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**Constructions of equality
in
a mathematics classroom**

Mike Ollerton BA, Adv. Dip. Ed. (Open)

**Thesis offered for the degree of
Master of Philosophy
of the
Open University
in the discipline of
Mathematics Education**

December 1997

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My thesis has been written over six years, and during this period I have worked with numerous pupils, students and colleagues in schools and in higher education institutions, at InSET courses and with acquaintances at many conferences and research seminars. When I first began my research I was teaching at Orleton Park School in Telford; here I received strong support from my headteacher Peter Hampson to develop mixed ability teaching; I therefore offer him my gratitude.

Since September 1995 I have been supported by my employer, the University College of St Martin, Lancaster, to complete my work. In particular I have gained encouragement from my Head of Department, Philip Gager; he also proof-read my final draft, and my gratitude similarly goes to him.

I am also indebted to Kath Cross, HMI for casting an inspectorial eye over an earlier draft and suggesting places where more rigorous argument would give my work greater significance.

Having the support of my family has, of course been crucial. It was my partner Ali Cooper who first suggested I study with the Open University in 1983 and work towards a BA.; this was followed by the Advanced Diploma. My work with the O.U. and this thesis has, therefore, been an evolution from that time; as such my deep love goes to her.

Christine Shiu was responsible for planting the first seed which led me to carry out this research and ultimately write my thesis. She fostered in me an initial belief that I had a story worth writing at a higher degree level, and the potential capability of being able to research and write it. Over the period of its production she has achieved the remarkable feat of only once displaying her frustration at my supreme ability to go backwards. In the main, Christine has quietly and calmly helped me steer my direction and make sense of the new horizons which frequently came into view.

Anne Watson, as my 'local' supervisor, has offered me great strength, in supporting my beliefs, during times when mixed ability teaching was (and continues to be) seen as an impossible or unfeasible goal to achieve. At the same time Anne has challenged and queried my explorations, seeking to ensure that the sense I sought to describe, of the experiences I had, was consistent with the directions my work took me. She has helped to give shape to my work and without her constant support my work would have been less substantive.

I therefore extend debts of gratitude to Christine and Anne.

Abstract

This thesis is a personal research journey, made through reflective anecdotes about notable events in classrooms. Explicit statements of my values as a teacher emerge from this analysis of my practice.

There are four main chapters:

Classroom Atmosphere identifies elements inherent in my interactions with children;

Curriculum Development accounts for the change in my pedagogy from a scheme-based to a problem-solving approach;

Teaching and Learning examines my perceptions of the different ways children learn mathematics and the types of strategies I have developed to support their learning;

Issues on working with un-settled groups provides a historical perspective on my mixed-ability teaching and explores my rationale in differentiating children's learning outcomes through accessible starting points and a range of extension tasks.

Throughout my research I pay attention both to children's academic and emotional development, and consider how issues relating to the content of the curriculum, to differential rates of learning, to how learning is fostered, and issues of non-selectivity of children within learning groups are inextricably linked. A key task has been to show the interconnectedness of issues, whilst demonstrating my understanding of each as a separate issue.

The evidence from my research demonstrates that it is both feasible and valid to provide children with their entitlement to equal opportunities in learning mathematics without separating them into pre-determined ability groups which are self-fulfilling, limiting and inherently discriminatory.

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Chapter 1

Introduction

My research focus

My research describes a personal journey of change. I analyse and synthesise the development of my pedagogical values as a teacher of mathematics. My aim is to reveal how I plan, carry out and evaluate my practice to achieve the ends my values demand. I construct an explicit rationale of these values which, in turn, underpins how I teach and encourage learners to learn. This rationale is forged by my experiences of working with pupils, students and colleagues in schools, higher education institutions and the wider mathematics education community.

My initial steps in research occurred as a result of reflecting upon experiences and events from lessons which I thought were interesting and significant. These events were written as anecdotes, and became a starting point, an access route for my research. I subsequently analysed and made explicit the issues and ways of working implicit within these anecdotes. Synthesizing the issues informed my theories on teaching and learning; over time, these theories became inseparable from my values.

In constructing this thesis I have become more aware of my practice; the range of strategies I use and how I interact with pupils and students. I have identified four main areas which encompass my research; these constitute my four major chapters:

Classroom Atmosphere: analyses the elements of my teaching which construct and nurture a positive learning environment;

Curriculum Development: analyses the processes of change which occurred in the nature of mathematics teaching I constructed;

Teaching and Learning: examines how my theories of teaching and learning evolved and analyses the strategies I use to guide and support learning;

Issues on working with un-settled groups: constructs a rationale for making positive choices to optimise pupils' access to equality of opportunity for learning mathematics.

The issue of equality, in a classroom context, emerged as a central theme; I define equality as each pupil's entitlement to optimise their learning of mathematics. The quotation below, from Plowden [1967:505], illuminated my thinking and encapsulates many of the issues I have explored. I have identified ten phrases (in bold type) and explore these within different chapters as follows:

- (i), (ii), (ix) and (x) in Classroom Atmosphere
- (vii) and (viii) in Curriculum Development
- (v) and (vi) in Teaching and Learning Mathematics
- (iii) and (iv) in Principles for working with un-settled groups.

A school is not merely a teaching shop, it must transmit values and attitudes (i). It is a community in which children learn to live first and foremost as children and not as future adults... The school sets out deliberately to devise the right environment (ii) for children to allow them to be themselves and to develop in the way and at the pace appropriate to them (iii). It tries to equalise opportunities (iv) and to compensate for handicaps. It lays special stress on individual discovery (v), on first hand experience (vi) and on opportunities for creative work (vii). It insists that knowledge does not fall neatly into separate compartments (viii) and that work and play are not opposites but complementary (ix). A child brought up in such an atmosphere (x) at all stages of his education has some hope of becoming a balanced and mature adult and of being able to live in, to contribute to, and to look critically at the society of which he forms a part.

The process of writing as a vehicle for reflecting upon my teaching aided the articulation of my values. Over time I became more selective about the anecdotes I wrote, choosing only to record and analyse events which offered new insights¹.

¹ Anecdotes are printed in different type-face, and are in a frame. Following each anecdote is a commentary about the significance of the issues which arose.

A personal journey over changing horizons

I begin by describing significant events in my childhood, about taking my 11+ and being a pupil in a Technical school². I continue this journey through my training, to becoming a teacher and then a teacher trainer. I have related events from my childhood in order to explore how these formative experiences shaped my values. Although some events occurred over thirty years ago, certain memories are vivid and remain in sharp focus.

Being failed by the 11+

I remember sitting in the examination room, terrified of the thought of failing and going to one of the secondary modern schools where, I understood, all kinds of terrible things happened. My elder sister had attended the Girls' High School and my elder brother was already at the Boys' Grammar school. And anyway, many of my friends had been told they would be given a new bike if they passed.

I remember fearing to look this way or that; a vivid memory was of a giant of a man telling the assembled testees that under no circumstances must anyone turn over the page until he tells us to do so.

As soon as the instruction was given I began. I was no 'slouch' and I completed the first two pages with ease; then I waited, and I waited, yet nobody seemed to be telling me to turn over to the next page... I continued to wait and there were just a few minutes left when I noticed an instruction at the bottom of the page which read:

Turn over to the next page

² Burnley Education Authority operated the tri-partite system of Secondary Modern, Technical and Grammar schools.

Contrary to others' expectations I didn't, therefore, go to the Grammar school. Perhaps a saving grace was that I had done enough on the Story and Comprehension test to qualify me to go to the Technical school.

Issues of fear are raised here; of passing and failing, of making simple mistakes under the pressure created by a system based on testing, and of the effects these mistakes can have upon individuals' aspirations.

This fear was to have a profound influence upon my development as a teacher; as a result of being failed by the system, I sought, through my interactions with pupils, to boost their confidence, to create an environment founded on the pleasure of learning, and to reduce the fear of learning mathematics.

Being in the bottom stream

September 1960 saw me dressed in a bright blue uniform, ready to begin my secondary education, at Burnley Technical High School. I was placed in 1D, the *bottom* class for everything. I knew I wasn't *stupid* but increasingly I accepted a bottom set mentality; and would frequently be found smoking with the 'toughest' of the 4th years boys behind the boiler house.

In lessons I tried hard to show I wasn't *all that stupid*, and seemed to come close to the top in tests. I remember feeling however that even if I did come top rather than third or fourth, this would only get me into the C stream whereas the *really clever* people were much higher up in the B and A streams. All hope of aspiring much beyond class D appeared futile.

after the 2nd year as this was not considered appropriate for set D pupils.

In my 4th year I opted for the more academic General (G) course. This was in preference to either the Building or Metalwork based courses; these more practical subjects were not my forte. In the G class however I felt enormously unconfident; everyone else was from the A or B classes. I felt a phoney, a fake who had no right to be mixing with these seemingly *brighter* children.

Being placed in the bottom stream as an 11 year old, and the effect this had on my self-esteem, behaviour, confidence and self-perception, were significant and powerful influences on my decision to become a teacher. This under-pinned the development of my values and the recognition of my responsibilities for providing all pupils' with equal learning opportunities.

'Good' and not so good teachers

During these early years I had some unpleasant encounters with teachers that caused me to fear several of them³.

Fortunately there were others who showed real care, who shaped my future and whom I subsequently adopted as role models. Mr Lambert was a brilliant Art teacher who enabled me to laugh whilst learning Art History⁴. He must have been good because I even forgave him for supporting Blackburn Rovers⁵.

In retrospect Mr Lambert was probably the first teacher to cause me to closely analyse something, in minute detail, in order to create meaning.

³ A particular event stood out when the woodwork teacher hit me across my hand with a steel ruler, for accidentally cutting myself with a chisel; he then sent me to the headmaster who in turn caned me.

⁴ Mr Lambert's skill at providing me with an enthusiasm for Art History was such that thirty years after these events, when I visited the Uffizi in Florence, I was able to take enormous pleasure from paintings he had encouraged his pupils to analyse such as Paolo Uccello's 'The Battle of San Romano' and Sandro Botticelli's 'Primavera'.

⁵ The rivalry between Burnley F.C. and Blackburn Rovers F.C. was 'serious' and intense.

Mr Green was a P.E. teacher who occasionally taught mathematics. He was obviously adept at teaching bottom sets, and I felt he showed some belief in me.

Upon deeper reflection, it was Mr Green who provided me with access to mathematics; he was the person who offered me the joy of real success and the confidence to feel I could do mathematics.

I was in the third year. One day Mr Green taught the class something markedly different from the kind of mathematics we had previously been used to: solving simultaneous equations. I didn't know why I was solving them or what they were for, but I knew this algebra work, which involved x 's and y 's, was what pupils in the top set did. I therefore felt to be doing something *really* important.

Discovering that I was capable of doing the kind of work which other pupils in higher sets did was a significant realisation.

Furthermore, I could use algebra to check my answers and know whether I had worked them out correctly; I remember the power of being able to do some *really hard* maths. That evening, quite voluntarily, I solved fifty or sixty simultaneous equations, filling page after page of my exercise book with their solutions. The next day I could barely wait for the maths lesson to begin so that I could show Mr Green what I had achieved. I was bursting with pride.

I had achieved success at a level beyond the expectations of myself and my teachers. This was my first encounter with the entitlement curriculum.

There was also a Chemistry teacher, who made learning fun and, with whom, many of my friends in the 6th form, including myself, fell in love. Mr Lawson was my Physics teacher; he also took school trips and organised weekend walks⁶. He moulded my

⁶ Mr Lawson also organised memorable extended walking holidays in Wales and Eire.

passion for hill-walking and climbing. It was ironic, therefore, that I grew to despise Geography. This was, in part, due to being publicly ridiculed in front of the rest of the class when the teacher read out an answer I had written in a Geography exam.

It does seem fairly amusing now, but at the time having one's stupidity revealed before one's peers, because I wrote about catching 'Kippers' in the North Sea, did not help me to become a confident geographer. Learning first hand about the adverse effect that ridicule can have upon a person's confidence to learn was a salutary experience; as a consequence this became a weapon I choose not to use. Those teachers who sought to nurture my interest in learning and a belief that I could achieve, despite the school system which offered me bottom stream status, were to have a strong influence upon the values I was to develop.

Collecting O levels and failing A levels

By the end of my 5th year I passed two GCE O levels, Art and Mathematics. I repeated my 5th year and eventually passed sufficient O levels, including Physics and Chemistry, to *scrape* into the 6th form. I had not at that time passed English Language and as I had *only* gained four grade 6's and one grade 4 (in English Literature) my confidence and self-esteem were still fairly low. At the 4th attempt I passed English Language with a grade 6. I took Chemistry and Mathematics at 'A' level.

Alan, my closest childhood friend, had attended Burnley Grammar school. He was *very good* at Maths and tried hard to help me, partly because he wanted to become a teacher⁷. In the sixth form, my struggle with academia continued. I failed both my 'A' levels, although this was in part due to other claims on my time which I describe below.

⁷It was ironic, therefore, that in 1969, having gained a Cert. Ed. from Clifton College of Education, Nottingham, he spent one term in a school in Essex then left teaching forever, complaining there was too much social work and insufficient scope for teaching mathematics.

Failing my A levels was to have an enormous influence upon my confidence as a mathematician; it impacted on me as a trainee teacher and through my early years as a teacher.

Other formative experiences

Throughout my secondary schooling I worked on my brother-in-law's milk round. I worked Friday evenings collecting money and at weekends and every holiday delivering milk. It was during this time I began to realise I liked people and customers seemed to like me. I was a *cheeky scoundrel* and yet it was the *grumpiest* customers, whom no-one else wanted to serve, who I seemed to charm with a grin and *close to the bone* comments.

Here I learnt the importance of befriending people to put them at their ease and the value of removing barriers. These became fundamentally important skills I employed as a teacher.

This work gave me much mental arithmetic practice; the power I felt in being able to work out customers' bills never left me.

Being able to mentally calculate the price of several different items and speedily work out the correct change gave me great pleasure. Here was a real life context which offered me success and through which I grew in stature; here my inadequate opportunities at school were compensated for.

My lack of success at 'A' level was compounded by working up to five nights a week as a waiter at the Burnley Cabaret Club. Here, however, I deepened my sense of self-belief; I had responsibilities and some customers would only be served by me. I was also, comparatively, earning a lot of money.

I do not know, at this stage, why I choose to train as a teacher; up to this point there had, seemingly, been no strong forces to encourage me to follow this path. However through my wider experiences I knew I wanted to work directly with people. A further sub-conscious force may have been a desire to emulate the role models certain teachers had provided; on this I can only speculate.

Training to be a teacher

I was accepted at Bishop Grosseteste College, Lincoln on a Primary/Secondary Certificate of Education in Mathematics course. This was on the strength of having followed an A level course, the shortage of mathematics teachers, and because the College wished to recruit male students, having previously been all female.

On this flimsy criteria I began my career as a teacher of mathematics.

The mathematics department, led by Mr Hawkyard⁸, was excellent although, because I had not passed A level, I struggled with several aspects of the course. An interesting and completely new aspect of learning mathematics involved exploring problems then writing up our findings.

In 1968, I first engaged with an investigative approach to learning mathematics.

From the outset I loved teaching. In my third year I had to choose between primary or secondary and, although I had thoroughly enjoyed my primary teaching experiences, despite having fifty two children in my class, I opted to do another secondary teaching practice.

I completed three very happy years at Lincoln and gained a Certificate in Education, which qualified me to teach either secondary mathematics or work as a primary teacher.

The possibility of doing a fourth year to gain B.Ed status, without having A-level mathematics, was never a serious option.

By now I knew I wanted to be a secondary mathematics teacher, and I looked for employment near the Lakeland hills.

⁸ Although Mr Hawkyard seemed an austere person, he also had a sense of humour; when decimalisation occurred he changed his name for the day to Mr Hawkmetre!

First teaching post

Not having immediate success at gaining employment in a secondary school, I took a post at Lowca primary, a 100 pupil strong school in a wind-swept West Cumbrian village. I worked there for two happy years.

Here I taught in un-setted groups; I had to find strategies for teaching children with different motivations and a wide range of learning potentials, across several subject areas. However, despite enjoying primary teaching, when an opportunity occurred to teach at Wyndham school, an 11-18 comprehensive, I pursued my earlier desire to work in the secondary sector.

Becoming a 'bottom' set expert teacher

At Wyndham, in 1973, pupils were setted in mathematics at the end of their 1st year. My first lesson was with a *bottom set* 2nd year group and although three pupils were all but out of control, I somehow survived. Whilst I occasionally taught a 3rd Year *top* set group, my skills were with 4th and 5th year *bottom set* pupils; indeed I prided myself on being able to motivate and develop strong relationships with these pupils. I consequently became a *bottom set* expert.

There were issues here of concentrating on what I felt expert at and in developing my skills accordingly; because my vision was limited by my confidence to engage with higher level mathematics, my potential as a teacher was restricted.

In my fourteenth year of teaching, having completed the Open University M101 course, I taught my first O-level class.

These experiences fuel my belief in the paramount importance of teachers being offered professional development opportunities and to engage with their subject at a challenging level; this requires time, encouragement and funding. This is of importance not only for teachers' individual development, but also for informing our understanding of ways children learn and develop.

Developing problem solving teaching approaches

During my twelve years at Wyndham I was strongly influenced by Eric Love, my Head of Department. I found him inspirational; he also brought a problem-solving approach to the department. He was interested in teaching in mixed ability groups and adopted materials for use with 2nd year pupils. The scheme was based upon the South Nottinghamshire Project and other ideas which Love gained from his involvement with the Association of Teachers of Mathematics. He was a driving force behind Mode-3 'O' level and CSE syllabuses in the mid-70's.

At Wyndham there existed a pioneering spirit to teaching and curriculum development. Coursework, or 'investigations', as we called them, became the vehicle for the teacher-assessed element of O level and CSE courses; they also began to influence ways of working in the lower school, where both standard investigations, and investigative approaches⁹, were developed.

Further inspirations

In 1983 I first encountered the teachings of Dorothy Heathcote¹⁰ as a consequence of my partner having studied with Heathcote for part of her PGCE and later for a higher degree. Heathcote brought a dozen or so of her MEd. students to Wyndham and used drama as a learning medium across various parts of the curriculum, including mathematics. Heathcote also organised after school sessions. I was also fortunate to attend one of her teaching sessions in 1985 where she asked her class to closely analyse a small part of a scene from Othello.

Although my contact with Heathcote was minimal, she had a significant impact on my thinking and practice. I adopted her methodology of closely analysing the essence of concepts I intended my pupils to develop and adapted this technique in my lesson planning.

⁹ The letter 'g' in this format denotes a word or phrase which appears in the glossary.

¹⁰ Then Senior Lecturer in Education (Drama), Newcastle-upon-Tyne University.

Becoming a head of department

In January 1986 I became the Head of Mathematics at Orleton Park, a North Telford school with 600 pupils on role. Prior to my appointment the department had purchased the SMP 11-16 booklet scheme; the structure within the department was mixed ability in the 1st year and setting thereafter. Following my appointment I was given virtual carte blanche, by a supportive senior management team, to develop wider teaching strategies.

In brief, the philosophy of the department changed, over five years, from setting to teaching in mixed ability groups; this change took place simultaneously with moves away from an over-reliance upon text books and narrow-focused worksheets to a more holistic, problem-solving pedagogy¹¹.

Studying with the Open University and early research

In 1984 I studied M101 with the Open University, the following year M245 and M203 courses. In 1986 I suspended my studies to concentrate on my head of department duties. I returned to the O.U. in 1989, taking EM235 and ME234 courses. In 1990, having achieved a B.A. I completed the E802 Advanced Diploma.

Gaining my first degree at the age of 42 was a defining moment. Through E802 I carried out Action Research; this proved to be an important catalyst for developing a wider range of teaching strategies to accommodate the move towards teaching groups Y10 and Y11 pupils in un-setted groups¹².

Becoming an initial teacher trainer

For five years from 1990 I worked part-time in initial teacher training in three higher education institutions, Keele University, St Martin's Lancaster and Manchester University. During this time I continued as a part-time head of department at Orleton

¹¹ This is fully documented in the Curriculum Development chapter.

¹² Developing strategies for teaching 4th and 5th year pupils in mixed-ability groups was the focus of my action research.

Park. During the 1993-94 academic year, my work-load was: two days at Orleton Park, two days at Keele and one day at St Martin's in Lancaster. I was therefore usefully placed, if rather thinly spread¹³, to put my theories into practice in initial teacher training. In September 1995 I was appointed full-time to the Mathematics Department at St Martin's Lancaster.

Summary

In writing this brief autobiography, I have reminded myself that, despite earlier negative educational experiences, I overcame assaults on my self-esteem and developed the drive to become a teacher, a head of mathematics and an initial teacher trainer. These experiences forged my belief in the value of constructing learning environments based upon positive teaching, and equality of opportunity, where all children can develop a sense of self-worth and achieve success.

By analysing these experiences, I have explored my practice in depth and have formulated the following conclusions; they are:

- denial of opportunities and challenge through judgements made on flawed evidence has profound, long term effect on individuals' future developments;
- by their attitudes, expectations and behaviours, teachers have a significant impact upon children's achievements;
- school and classroom organisational imperatives have prophetic impacts upon pupils' and teachers' self-fulfilment and self-esteem;
- teachers and pupils alike require positive input, support and encouragement to drive and challenge personal expectations;
- manifestations and possibilities of success nurture the desire to learn;
- mathematics constructed as a problem-solving, investigative discipline fosters open-ended enquiry, and enables learners and teachers to fulfil and challenge their own expectations.
- to pre-judge what children will achieve in the future, based upon what they have achieved in the past provides an unsafe basis for teaching and learning.

¹³ A further call on my time was the role of ATM Easter conference officer in 1993 at St Martin's Lancaster and in 1994 at Ripon & York St John.

Hypothesis

My hypothesis is: although teachers' practices are idiosyncratic and different, significant benefit for teaching and learning can be gained by enagaging in common, self-reflective, analytical processes. By noticing, in-the-moment, and recording anecdotes of events from lessons, teachers can reveal to themselves, and to others, principles upon which their current practice is based. This is a valuable precursor for debate, in order to construct a commonality of principles and values, and to underpin and further develop an effective pedagogy of teaching and learning, based upon informed practice.

Through the research of my practice, I provide a model, a process of thinking and self-analysis which is transferable to other teachers' practice. This process of writing anecdotes about classroom events and evaluating the issues which arise has enabled me to find my truths behind the way I work and to discover the principles fundamental to my practice. In turn this generates a further, deeper understanding of my rationale; it refines my pedagogy and defines my values; the process is continuous.

Chapter 2 Methodology

Outline

This chapter's sub-sections, describing and discussing my research methodology, are summarised below under their corresponding sub-headings:

- **Aims and intended outcomes of the research**
- **Locating my research:** discussing research methodology and rationalising the action research method I used.
- **First and final titles for the thesis:** considering how my final title emerged in light of my research processes.
- **Beginning to research my practice:** giving an account of the contexts from which the research emerged.
- **Writing anecdotes:** describing the research process chosen to analyse significant events in my, and others', classrooms.
- **Defining strategies and principles within my teaching:** recognising significant issues and identifying them as strategies and principles.
- **Classifying and clumping issues:** analysing and synthesising issues arising from the anecdotes.
- **Finding analogies:** describing and analysing wider experiences beyond the classroom which informed the research.
- **Siting my research process:** The Discipline of Noticing.
- **Beginning to construct theories about teaching and learning:** making explicit the values I implicitly held.

- **Theoretical frameworks:** using and building on existing theory.
- **Inter-connectedness of issues:** describing the process of isolating issues which are inextricably linked.
- **Semantics and the power of language** discussing uses of terminology.
- **Reading as part of the research process**
- **Concluding remarks**

Aims and intended outcomes of the research

To maintain integrity and effectiveness in one's practice, I believe there is a need to keep revisiting the question, "Why am I doing this?" This applies equally to research and teaching and to the ways in which learners are encouraged to think. Running parallel, therefore, to the question "Why teach?", I began this project with a need to ask the question, "Why research?" Writing this thesis became a vehicle to address these questions.

Throughout my research I have been fully engaged in the processes of teaching and have carried out two distinct roles; researcher-self and teacher-self. At moments of *spective*¹ awareness, when I consciously responded *in-the-moment*², these roles co-existed simultaneously. Such times apart, I have built my research upon retrospective reflections of events from my classrooms and, to a lesser extent, in PGCE students' classrooms. Writing anecdotally about certain events, and subsequently identifying and analysing issues which arose, formed a central part of my work. This process enabled me to sharpen my perception and raise my ability to articulate the implications arising from them; this articulation is an explicit recognition of my values which, in turn, I take into my classrooms and my teaching.

¹ I define *spective* as follows: Retrospective is in the past tense; prospective is in the future tense; *spective* is the present tense.

² I develop this issue of 'Noticing' later in this chapter, p. 29.

Locating my research

Let not your first thought be your only thought.

Think if there cannot be some other way.

Surely, to think yours own the only wisdom,

And yours the only word, the only will,

Betrays a shallow spirit, an empty heart³.

Sophocles.

Central to the development of effective practice is the need to research, rationalise and construct common principles, to debate values which underpin pedagogies of teaching and learning. Whilst individual teachers' practices are unique and to an extent idiosyncratic, it is essential to engage in debate and share pedagogies in order to try to reach consensus. A pre-requisite to this is the commitment to explore and reach an explicit rationale of one's own practice. I sought to achieve through reflecting upon my practice. Stenhouse [1975:144] describes this as *extended professionalism*, which he defines as:

The commitment to systematic questioning of one's own teaching as a basis for development;

The commitment and the skills to study one's own teaching;

The concern to question and to test the theory in practice by the use of those skills.

Commitment to reflective practice is echoed by Kemmis [1993:182] as:

Practice, as it is understood by action researchers, is informed, committed action: praxis. Praxis has its roots in the commitment of the practitioner to wise and prudent action in a practical (concrete historical) situation. It is action which is informed by a 'practical theory', and which may, in its turn, inform and transform the theory which informed it. Practice is not to be understood as mere behaviour, but as strategic action undertaken with commitment in response to a present, immediate, and problematic action context.

³ Haemon beseeching his father, Creon, to reconsider the death sentence placed on Antigone.

Through systematic reflection upon my teaching, I sought to devise a research method for processing and self-analysing my thinking; to construct a 'practical theory' and develop my practice. This model generated a deeper understanding of my rationale and served to further refine and redefine my pedagogy. This process forms a continuum: *In terms of method, a self-reflective spiral of cycles of planning, acting, observing and reflecting is central to the action research approach.* [Kemmis 1993:178].

My research method evolved to form the following processes:

- systematically describing my practice;
- analysing my practice and identifying practical principles and strategies;
- seeking to construct theoretical principles;
- making these principles explicit.

By articulating my truths I entered into...*argumentative discourse leading to rationally motivated consensus.* Hirst [1993:159]. I began to see this as an appropriate model for other teachers to actively research their practice, extend their professionalism and enable rationale debate and consensus about effective teaching approaches.

Alternative research methodologies

To examine effectiveness in my teaching practices, there existed three possible choices, these were:

- (i) for others to examine my practice;
- (ii) for myself to examine other teachers' practices;
- (iii) for myself to examine my own practice.

Either of the first two approaches would have enabled me to draw conclusions about the relative effectiveness of teaching practices. With regard to (i), I welcomed outside researchers entering my classroom; indeed I was frequently visited and observed by many teachers, lecturers and students⁴, and I entered into

⁴ Two students who researched my practice were: Boaler, J. (1991) Challenging 'School' Mathematics: The Integration of Process and Content, Context and Culture, Dissertation, submitted for M.A. degree, King's College, London University; and Eales, A. (1991) Styles of Effective Heads of Mathematics in Secondary Schools, submitted for Pd D Thesis, University of Leicester.

discussions with many people about my practice. However, as each visitor had their own specific agenda this did not provide a systematic exploration of my work. Furthermore, my initial interest was to explore my own practice, to see this through my own perspective, rather than have an outside researcher filter observations through their terms of reference. In addition to this an outside researcher would not have been able to achieve the same degree of intensity or give the equivalent commitment of time and focus to my work.

With regard to (ii) I was interested in first understanding and articulating my practice compared with others, about which I refer later in this chapter⁵. Additionally, systematically researching others' practices was not feasible, given the constraints of my varied work patterns⁶. However, during my research I have observed other teachers and trainee teachers and have described and analysed the practices I perceived⁷.

My research comes into the methodological category of action research:

Action research is a form of self-reflective enquiry undertaken by participants in social (including educational) situations in order to improve the rationality and justice of (a) their own social or educational practices, (b) their understanding of these practices, and (c) the situations in which the practices are carried out.

Kemmis [1993:177].

All research methods have inherent strengths and weaknesses; I felt the advantages of action research outweighed its shortcomings. I explored the potential of the method with reference to Hammersley [1993:218]; below I have summarised this debate with the use of selected quotes to illustrate my context and intentions:

⁵ See 'Beginning to research my practice' pp. 22-24.

⁶ See Introduction p. 13 'Becoming an initial teacher trainer'.

⁷ See 'Classroom Atmosphere' chapter pp. 72-75, 'Curriculum Development' chapter pp. 110-111 and 'Un-settled' chapter pp. 179-182.

Advantages	Disadvantages
<p><i>... teachers have access to their own intentions and motives, thoughts and feelings, in a way that an observer does not...</i></p> <p>By researching my practice I gained access, explicitly, to my intentions and motives.</p>	<p><i>People can be wrong even about their own intentions and motives... Also, understanding often requires seeing a phenomenon in its wider context.</i></p> <p>My motives were to look for effectiveness in my practice rather than construct notions of right or wrong intentions.</p>
<p><i>... the teacher-researcher will usually have long-term experience of the setting being studied... as well as other information that may be required to understand what is going on.</i></p> <p>My long-term experience was twenty years teaching prior to beginning my period of research. I therefore, potentially, had access to a breadth of information.</p>	<p><i>The information that practitioners have about the situations they operate in is a product of experience deriving from a particular role that will have given access to some sorts of information but not to others. In particular their understanding... may be superficial or distorted. An outside researcher may be able to tap a wider range of sources of information than an insider...</i></p> <p>Systematic research involved me in reviewing theory and others' practices which served to deepen my understanding of my practice.</p>

Advantages (continued)	Disadvantages (continued)
<p><i>... the teacher already has relationships with others in the setting... an outsider would need to spend considerable time... building up such relationships.</i></p> <p>The relationships I forged with pupils and colleagues had a profound effect upon certain aspects of my research.</p>	<p><i>... the relationships available to the practitioner will exclude as well as include, and may not include what is necessary for research purposes.</i></p> <p>Just as the practitioner is affected by the relationships s/he has with pupils and colleagues, so will an outside researcher be similarly affected.</p>
<p><i>... teachers are the key actors in the settings studied... they are in a position to test theoretical ideas in a way that a mere observer can never do.</i></p> <p>This was intrinsic to my work and consequently formed the basis upon which I was able to develop my research.</p>	<p><i>What is required to test theoretical ideas may well conflict with what is needed for good practice... The practitioner may therefore, and... may not be able to test his/her ideas.</i></p> <p>My concern was to develop <i>good practice</i>, and my intention was, therefore, to use my research to achieve this. It was inappropriate to test theoretical ideas in isolation, as I wanted to analyse the impact of the contextual factors. I needed therefore to employ an operational research method that enabled concurrent reflection and practice.</p>

I make comments in my conclusion to this chapter about the ways in which future research could further validate my findings.

First and final titles for the thesis

During my research I have engaged with a number of issues which either challenged or confirmed my belief in the value of teaching children mathematics in un-settled groups. At the time of beginning my research, the central issues I worked on as a teacher and as a head of department were planning lessons, constructing modules and devising schemes of work based upon problem-solving approaches, and teaching pupils, across the 11-16 age range in un-settled classes. An early working title which emerged, therefore, was: "Ways of working with mixed-ability groups in the upper school".

As a novice teacher trainer such developments had wider implications for my work with students and the methods they were encouraged or discouraged to use in their school placements. This meant I had responsibilities which went beyond those relating to how I taught within the confines of my classroom. As my research progressed I recognised the issues behind working with un-settled groups, the strategies I used, and the rationale underpinning this were only part of a broader picture of my philosophy of teaching. This philosophy, emerged, in part, as a result of attending ATM⁸, BSRLM⁹, and BCME¹⁰ conferences and OUCME¹¹ research days and seminars. This broader picture was one of equality; of children's entitlement to learn the same mathematics, and my role in providing such opportunities. Consequently my focus shifted and my title changed to: "Constructions of equality in a mathematics classroom".

Beginning to research my practice

A number of events and situations provided me with opportunities to reflect upon my practice in teaching mathematics. I found myself noticing different emphases I placed on my practice in comparison with many other teachers. This drove me to want to research and analyse events from my own classroom work and I saw myself as being in the best position to do this: *...only the practitioner can study*

⁸ Association of Teachers of Mathematics.

⁹ British Society for Research into Learning Mathematics.

¹⁰ British Congress of Mathematics Education.

¹¹ Open University Centre for Mathematics Education.

praxis. Action research, as the study of praxis, must thus be research into one's own practice. Kemmis [1993:182].

Opportunities for reflection included:

- moving from a 'progressive' department, which had developed the use of investigative approaches since 1973, to a more 'traditional' department in 1986, where the predominant approach was didactic and exercise driven;
- discussions with other teachers at local GCSE coursework meetings;
- discussions with other heads of mathematics at county-wide meetings;
- disseminating my practice through InSET;
- discussions with other teachers at ATM conferences;
- discussions with other teachers at ATM General Council meetings;
- collaborations with other teachers using the ATM-SEG GCSE syllabus;
- working with ITT students in a number of schools¹² both as a tutor in three HE institutions and as an external examiner for a further two.

An early process of my research was to list elements in my practice, (left hand column), and compare these with practices I discussed with other teachers and observed in their classrooms, (right hand column¹³).

investigative, problem solving approaches integrated into schemes of work	investigations most frequently used as bolt-on tasks, separate from the main curriculum
a modular scheme of work constructed to create holistic ways of learning	skills taught in fragmented, atomised ways
practical equipment frequently used	practical equipment rarely used
classroom furniture frequently rearranged in a variety of ways	classroom furniture rarely moved

¹² Since 1990 I have visited about 50 schools.

¹³ I do not wish to imply that the practices I describe in the left hand column were unique to my teaching; I also recognise some of the practices in the right hand column were those I had sometimes adopted as an early teacher, although far less frequently in the last ten years.

text books and repetitive types of worksheets never used	textbook and worksheets frequently used as the central resource
all children learning mathematics in un-setted groups across the 11-16 age range	vast majority of children learning mathematics in settled groups
the use of 'new' technology, such as graphic calculators (1990) with classes from Year 7 upwards	graphic calculators rarely used below Y12 or Y11
a coursework based GCSE syllabus devised in conjunction with the Association of Teachers of Mathematics and the Southern Examining Group, from 1986	many departments chose not to use coursework for assessment purposes at GCSE level, or chose to do the minimum required
'tick' sheets ¹⁴ were not used for recording the work pupils did in relation to National Curriculum statements of attainment	departments adopted some form of National Curriculum tick-list in a bid to record what pupils had learnt.
pupils encouraged to write about their mathematics from Y7 upwards	pupils experienced writing about mathematics usually when 'doing an investigation'
substantive feedback provided for pupils in written comments as part of the 'marking' process	the most common marking practice was to tick or cross pupils' work

Writing anecdotes

In order to be explicit about these different emphases in my practice, I began to write anecdotes¹⁵ from my lessons where I reflected upon interesting events which occurred in certain lessons. Telling stories about classroom events and

¹⁴ The alternative to not using a 'tick' sheet was to arrive at an overall impression of pupils' achievement, throughout the course of a year, according to the types of conversations with pupils and the work they produced.

¹⁵ The page numbers of the anecdotes and dates they were written appear in appendix 1a.

evaluating the issues which emerged enabled me to find my truths behind the way I work and to discover the principles fundamental to my practice.

The audience for these anecdotes were my supervisors; having an audience was important because sharing events helped me shape and contextualise them.

During this early stage I chose not to set myself any specific criteria for pre-determining what to record; I described what I found myself noticing. I did not attempt, at this stage, to link issues together, nor did I look for specific exemplars to illustrate a particular principle or strategy. However, as I reflected upon lessons, certain principles or recurrent teaching strategies began to emerge.

Finding ways of describing my practice, to identify and rationalise my principles and strategies, became a significant process in further understanding my practice. Whilst these were implicit within my practice, my research caused me to become explicit about them.

...our understanding of action is in large measure necessarily derived from an analysis of what is judged to be successful action before we understand, let alone formulate explicitly, the rules or principles that it embodies... Hirst [1993:155].

I noticed parallels between myself writing and becoming explicit about my principles and strategies, and pupils writing to 'fix' their learning¹⁶.

This process of reflecting upon events also helped me sharpen my conscious awareness of events as they occurred; later I found this way of working, of constructing meaning from noticing and reflecting, was consistent with Mason's "Noticing in the moment".

Only my awareness is educable, in the sense that my power to notice can be developed and refined, and my noticing can be focused and directed. Only when I notice spontaneously, for myself, can I choose. Only when I notice my self, do I become awake and free. Mason. [1987:30].

¹⁶ See 'Pupils fixing their learning' in the 'Teaching and learning' chapter pp. 136-137.

Within some anecdotes several interesting events occurred; at other times a single issue stood out. An interesting event was one which drew my attention to an issue about teaching and learning, where a strategy appeared to enhance pupils' learning. For example, the anecdote about Suzie's apparent difficulties of learning trigonometry¹⁷, was, at the time, an interesting event. My reflection upon how I responded in this situation, and my recognition of how her learning was linked to confidence, shifted the event from being interesting to significant. An interesting event, therefore, became a significant one through the processes of analysing and evaluating my, or others', responses to situations and the effect this had upon pupils' learning. This process of selecting certain events from anecdotes, of deciding what was interesting and how it became significant, and deciding where in my writing to record an event, became intrinsic to my qualitatively-focused research.

Articulating my understanding through writing caused me to pay close attention to events and formulate meanings; this, in turn, further shaped my learning. Reflecting upon events, was the precursor to analysis and synthesis of issues. Analysis helped me identify and make explicit the principles I held and the strategies I devised for supporting them. This process of being explicit about my principles and strategies created my pedagogy, and modified and refined my practice. This process was cyclical and, at its most efficient, spiral by nature.

Writing anecdotes therefore served the following purposes:

- keeping a record of significant events from my lessons;
- teaching myself how to focus on specific details within my lessons;
- learning how to write about my practice of teaching;
- creating curriculum development opportunities within the department;
- being explicit about events implicit in my teaching led me to construct theories about teaching and learning;
- raising further issues helped me to substantiate and test my operational theories about teaching and learning;
- constructing explicit theoretical, educational rationale of my practice enhanced the quality of my practice.

¹⁷ Atmosphere chapter pp. 61-63.

Defining strategies and principles within my teaching

Writing about my practice and analysing events from the anecdotes revealed a range of issues¹⁸. To identify the strategies I used and the principles implicit in my practice, which existed in my belief system, I annotated each anecdote. Later I numbered and coded each issue: P for principle and S for strategy for example:

- 1 - entitlement (P)
- 2 - trust (P)
- 3 - confidence (P)
- ...
- 25 - the "other" (S)

As I wrote more anecdotes, I began to see some issues recurring and new ones arising. This led me to a process of a process of classifying and clumping.

Classifying and clumping issues

To define the issues I produced a glossary of terms. This helped me to name and classify issues. Next I drew up a two-way table¹⁹ to connect the frequency of the issues with the anecdotes. My next stage was to clump the 40 identified issues. I did this by writing each one on a separate piece of paper and then grouped issues which appeared to be linked. For example 'trust' and 'confidence' were issues which I saw as linked values, and which permeated the atmosphere I intended to promote in my classroom.

Initially five clumps emerged:

- Classroom Atmosphere;
- Curriculum Development / Reflecting upon Practice;
- Ways of Learning;
- Teaching Strategies; and
- Issues on working with un-settled groups.

¹⁸ See appendix 1b.

¹⁹ See appendix 1c.

Each clump had sufficient substance to act as a focus for a separate chapter. However whilst writing each chapter, I kept challenging my thinking as to whether each issue might better fit into another chapter. This process led me, six months later, to test out my original clumping decisions. I decided to re-sort them, to see if I arrived at the same result. I wrote on the back of each 'issue sheet' my original classification²⁰, shuffled all the pieces of paper into a pile and re-grouped them. Of the forty pieces of paper, I re-classified thirty five of them in the same way as I had originally done. This confirmed the validity of my original classifications.

Constructing my theories was facilitated by this process of classifying and clumping issues; this proved a lengthy and complex process which may be disguised in the final product of my thesis.

Beginning to construct theories about teaching and learning

Whilst my theories about teaching and learning existed implicitly in my practice, they emerged as I became explicit about them.

Getting at current practice and policy will necessarily involve articulating accurately the concepts and categories that practitioners use implicitly and explicitly, for it is only from descriptions and principles formulated in these terms that an overt rational critique of practice is possible...(this is) what I shall call 'operational educational theory'. Hirst [1993:155].

As I recognised the emergence of certain theories I began to look for evidence of how I put them into practice. Specific anecdotes which I have drawn upon in the four main chapters, i.e. 3, 4, 5 and 6 are listed by page number in appendix 1a.

There are other anecdotes which I wrote between June 1991 and February 1994 which I have not included in the main text yet which helped me construct a wider

²⁰ On the back of each sheet clumped under 'Ways of Learning', I wrote WOL, etc.

perspective on my thinking, specifically during the process of recognising, and classifying and clumping issues.

Anecdotes which I collected pre-June 1991 and post-February 1994 exemplified and validated my recognition of a particular theory, however, they do not appear in appendices 1a and 1c, as they did not further inform the classifying and clumping process.

Through the process of filtering my writing, of drafting and redrafting I engaged with many experiences which I have chosen not to include in the main chapters of my thesis. I offer reasons for this below. However, because these experiences had an impact on my thinking and rationale I have included them in this chapter under the next sub-heading: "finding analogies".

Finding analogies

My research has dominated much of my thinking in wider experiences beyond those of the classroom, and by analysing some of these other, life-enriching experiences, I frequently drew comparisons between learning in these contexts and learning mathematics. These other contexts which provided me with analogies were:

- learning to juggle,
- watching a theatre event,
- watching a film,
- self-service shopping, and
- learning to climb.

They were not however sufficiently robust in themselves to transfer directly to a classroom situation. For example, learning to juggle is an esoteric pastime which only a small number of people choose to learn; mathematics, on the other hand, is taught to all children, without choice, as a lawful statute.

The analogies were useful as vehicles for drawing comparisons with principles common to teaching and learning. I briefly describe these contexts and the issues they raised.

Learning to juggle

When learning to juggle, I wrote an article²¹ where I drew three comparisons, these were:

- access for the learner by starting with a simple task;
- practising basic skills in the context of solving a harder problem;
- the inequity of providing different curricula to children in 'low' sets and the consequence of such children not being provided with opportunities to access the same curriculum as their peers in 'top' sets.

Watching a theatre event:

On seeing a play²² performed by a Russian theatre company and analysing their interpretation of a particular scene, I wrote about 'surprises'²³. This deepened my belief in the value of providing learners with surprises in mathematics to gain their interest and strengthen their understanding.

Watching a film:

On analysing a scene from a film²⁴ I drew analogies with assessment, by comparing the method used to gain information about a person's capability to perform a certain task, whilst restricting the conditions within which they carried out the task. These events had implications for recognising the complexities of measuring children's mathematical understandings through a simplistic testing regime and the dangers of using such results to objectively quantify children's ability and, therefore, to place children in sets accordingly.

I also drew comparisons using another clip from the same film to distinguish between knowledge and wisdom. This occurred as a result of recording a quote Mason had offered to distinguish between knowledge and wisdom: *Knowledge - knowing that, knowing how. Wisdom- knowing to*²⁵, when he invited the group to 'mark' an issue at an OUCME research day in June 1994.

²¹ Ollerton, M [1995a].

²² A Midsummer Night's Dream.

²³ Ollerton, M. [1995d].

²⁴ Butch Cassidy and The Sundance Kid.

²⁵ Kornfield, J. [1994].

Self-service shopping:

I used the analogy of a proprietor of a particular shop²⁶ who entrusted customers to weigh out and cost their own herbs and spices, and drew comparisons with issues of 'trust' in the classroom.

Rock climbing:

Re-learning the basics of rock climbing, initially at an indoor climbing wall²⁷, caused me to focus on dual issues of learning specific, basic skills and the importance of having problem-solving contexts within which to apply these skills.

Siting my research process: The Discipline of Noticing

Noticing begins as Retrospective Awareness, and hopefully, turns into Spective Awareness. If you notice in the moment, spectively, you have an opportunity to participate in choice. For me there is real freedom, in just that one moment; the moment when you are awake and alive to possibility. Mason [1991].

Developing my spective awareness, to notice more in-the-moment, grew as a result of reflecting upon events, and as a consequence of analysing and evaluating anecdotes. Noticing in-the-moment is a way of being which I have consciously chosen to work on; it is a skill I seek to develop further. My awareness has grown as a result of recording certain events from lessons. Although the anecdotes capture only fragments of events from lessons, they are ones which drew my attention and raised my spective awareness. I cannot, of course, constantly be in a state of spective awareness, and there have been moments of reflection when I wished I had been more spectively aware, in order that I may have responded differently during a discourse with a pupil, a student or a colleague. The value of reflecting upon events and seeking spective awareness, was powerfully illustrated to me whilst observing a lesson taught by a PGCE student in December 1995.

²⁶ Crabapple wholefood shop, Shrewsbury.

²⁷ Kendal climbing wall.

Jamie was working with a Y10 group. The pupils normally sat in rows and worked through a well known scheme aimed at GCSE Foundation level. He was successfully taking a number of *risks*²⁸ and was having an excellent lesson. This was based upon an exploration of area of shapes. The pupils had been encouraged to put their desks together and work in small groups. However because he was constantly interacting with pupils, he wasn't able to stand back and enjoy, for himself, just how well the lesson was proceeding. As the pupils were so thoroughly engaged in their work, I felt he could have taken 'time-out' in order to observe for himself what was happening.

I discussed this with Jamie in the post-lesson de-brief. The following week Jamie described how he had acted upon these discussions and had made a conscious decision to take time out during another lesson. He explained how revealing he had found this experience, how much he had learnt as a result of standing back and noticing in-the-moment.

The notion of noticing is critical in any profession, for if I do not notice, I cannot choose to act differently. If I am not sensitive to some feature, I can do nothing about it. So in order to develop my professional practice I need to be developing and refining my sensitivities, extending the things I notice about my practice.
Mason [1996:19].

Heathcote [1984:24], whose work I have also paid attention to, places equally high value on teachers being aware of the underlying importance of making choices; she relates this to the quality of teaching:

We make choices on excellence daily, minute by minute, each choice dictating the next. You can't reach excellence for a whole day; you can only reach it minute by minute. And this is one of the excitements of teaching - the constant exhilaration of recognizing the choice we have made at any moment. When we stop choosing, things go radically wrong with us.

²⁸ I develop issues of taking 'risks' in the Curriculum Development chapter pp. 106-111.

Theoretical frameworks

Teachers, Mathematics and Children:

As a result of giving a seminar at the November 1992 BSRLM meeting, I adapted the existing theory constructed upon the triad of Teachers, Mathematics and Children (T, M, C). My theory was that interrelationships between teachers, mathematics and children exist in whichever way teaching groups are formed, whether they are setted or not. However when issues of equality and entitlement are accounted for, I conjectured that a third dimension of mixed-ability (M-A) must be included; these issues are not being considered when children are placed in sets according to notions of mathematical ability. Thus the triangle labelled T, M, C, grew into a tetrahedron²⁹ with M-A at its apex³⁰.

Through my research I found other educationalists who endorsed the theoretical frameworks I was constructing; this helped me articulate my rationale which underpinned these frameworks. I illustrate this with reference to the following issues: the social and psychological aspect of childrens' learning, and my decision not to give prizes as extrinsic rewards, permeated my practice.

Children learning mathematics and their social and psychological development:

I considered whether my underlying rationale for teaching in un-setted groups was aimed at improving childrens' learning of mathematics or whether it was an attempt to provide educationally harmonious and social conditions for learning: *Mathematics is an inherently social activity*, Schoenfeld [1994:60]. I was drawn by a remark made in a 'Panorama' programme³¹ by Lin Lee Hau, a Taiwanese teacher who, in response to David Reynolds' question about why children are not separated into different ability groups explained: *Because it is not allowed in Taiwan. If it was, less advanced students would feel they are being labelled. It's like saying they'll never advance; their class mates would treat them differently. So this policy respects the children's psychological development.*

²⁹ See appendix 2 for an up-dated version of the triad growing to a tetrahedron.

³⁰ Ollerton, M. [1992].

³¹ BBC Panorama 'Worlds Apart' 5-6-96.

My conclusion on this issue is that children's academic and social development are inseparable and to engage in debate that one is more important than the other is counterproductive.

On not giving extrinsic rewards for achievement:

For many years I had questioned the ethos of constructing a reward system; of offering prizes or merit marks to children as a reward or as a reason for learning mathematics. I therefore considered alternatives. This was to be discussed with pupils their responsibility for their learning and the value of learning being its own reward. On this issue I was influenced by Bruner [1972:77]: *You can corrupt them all too easily into seeking your favour, your rewards, your grades ;* and by Donaldson [1978:115]: *The traditional way of encouraging children to want to learn things that we want to teach is by giving rewards for success: prizes, privileges, gold stars... The obvious risk is to the children who do not get the stars, for this is just one way of defining them as failures. The other risk is to all of the children - 'winners' and 'losers' alike. There is now a substantial amount of evidence pointing to the conclusion that if an activity is rewarded by some extrinsic prize or token - something quite external to the activity itself - then the activity is less likely to be engaged in later in a free and voluntary manner when the rewards are absent, and it is less likely to be enjoyed.*

Inter-connectedness of issues

By July 1995 my writing had moved, in the main, from modes of collecting anecdotes, analysing and making meaning, to conveying the outcomes of my research. I nevertheless continued to write occasional anecdotes from lessons. Whilst writing up I frequently encountered a problem of separating out issues in order to describe them as discrete elements, whilst recognising they were inextricably linked. To describe this constant dilemma I provide the following train of thought³².

³² The issues are written in bold italics.

With regard to childrens *equality* for learning mathematics, I believe all have common, basic *entitlements* to be offered *opportunities* to *access* the same concepts. To support this I offer common, *simple starting points* at the beginning of each *module* of work. Subsequently, because different children will develop ideas to different levels of complexity, I must take *differentiated* outcomes into account. I accommodate this by creating *story lines* for each module, which involves planning *extension* or *next* tasks. Through the construction of modules, I seek to devise a *holistic*, approach to the curriculum, where concepts emerge and skills merge. This *inter-connected* approach defines a methodology which contrasts to one where skills are primarily learnt as separate entities in *fragmented* ways. Within each module I expect to have a range of types of *teacher interventions* with pupils; this includes the different ways I may *answer* the same question from different pupils.

A significant responsibility is to be involved with *curriculum development*, to look for *problem solving* approaches, to provide *challenges*, to stimulate pupils to *explore* situations and, as a result,...

Later I realised this was the basis of a broader statement of my values; I develop this in my concluding chapter.

The problem of separating issues became even more difficult when I tried to write 'Ways of Learning' and 'Teaching Strategies' as discrete chapters. For example, when writing about children gaining access to concepts from a simple starting point, I also had to explain my role as teacher in setting up the starting point. I therefore decided to clump these two groups of issues together into one chapter.

Four main chapters finally emerged as:

- Classroom Atmosphere
- Curriculum Development
- Teaching and Learning
- Issues on working with un-settled groups

Semantics and the power of language

My research has been a potent instrument for examining and articulating my values. By examining the words and phrases I use, I raise my awareness of my intentions and uncover hidden values. In this section I briefly discuss two issues relating to terminology and my awareness of the language I use.

Mixed-ability and un-setted classes

Ability grouping, where it exists, may lower expectations and, may be unfair... If we challenge ability as an organising framework, however, it hardly seems appropriate to introduce the concept of 'mixed-ability' Both are tainted by the spectre of debilitating determinism and both appear to provide for a sterility in educational and political debate. Bourne and Moon [1995: 32].

During my research I re-considered my use of the terminology 'mixed-ability'. The notion of mixing can be perceived as another form of social engineering which to embark upon is as fraught with difficulty as it is to create setted groups. Alternatives, such as 'pastoral' groups, were suggested implying the teaching group was the same as the form or tutor group.

The case for non-streaming is as important today as ever... we can make a useful start by abolishing the unfortunate expression 'mixed-ability', the term is seriously misleading and makes non-selective grouping a target for detractors. Teachers of non-selective groups aim to encourage students to develop a wide range of abilities through appropriate learning experiences yet 'ability' in the singular suggests that each student has a single ability. In order to create a 'mix', one must presumably be able to measure, estimate or in some way identify it. The expression 'mixed-ability' appears therefore, to give credence to old discredited theories of intelligence. 'Mixed-ability' has also acquired connotations of social engineering when the reverse is the case; it is in-school selection, often based on the most dubious of criteria, which falls into that category. Copland [1993:81].

Applying any label may be construed as contentious; I prefer to teach groups of children who have neither been socially nor intellectually grouped together. I do not seek here to engage with 'political correctness'; I do intend to challenge setting as a phenomena which is divisive and undermines the educational aspirations of children and the fabric of schools which perpetuate such inequities.

Pupils and students

A discomfort I have felt throughout my thesis is my use of the word 'pupil'. Before working in initial teacher training I referred to those I taught in secondary schools as 'students'. This was because the word 'student' carried a more mature connotation relating to scholarly activity; 'pupil' offers a more subservient description of a beginner or a novice. However, in order to distinguish between learners in the 11-16 age range and those I work with on ITT courses, it became impractical to refer to school-students and ITT-students. On pragmatic grounds then, I have used 'pupils' when referring to those I taught in the 11-16 age range and 'students' when referring to initial teacher trainees.

My preference for using 'students' instead of 'pupils' is a minor outcome of my developing awareness; more significant is my distaste for the way some children are labelled by some teachers as "bottom set children", and even worse "thick" or "stupid".

Whilst the differences between 'mixed-ability' and 'un-setted', 'pupils' and 'students', may appear to 'only' be a matter of semantics, I have nevertheless become increasingly aware of the power of language and the messages which are transmitted as a result of the way language is used. In turn, the words we use, how we use them and the intention behind our use of words carries a great deal of power which affects perceived meaning and received understanding.

Reading as part of my research process

...I do not believe that reading any book is enough to change a person's practice, it is possible that such a reading may plant a seed in a person's mind which will grow, and eventually come to affect practice - provided always that it falls on hospitable soil and gains some support from a sustaining environment.

Sotto [1994:83].

Reading as teacher-self

In the main my reading followed on from my writing. As a teacher and head of department I read key reports, such as Cockcroft (1982), Mathematics from 5 to 16 DES (1985), National Curriculum Non-Statutory Guidance, and papers and journals including the Times Educational Supplement, the Education Guardian and, most significantly, Mathematics Teaching (MT)³³. MT became a major influence in helping me form opinions about teaching mathematics, providing insights into other teachers' practices and validating aspects of my practice. For example, Mason [1986: 28] describes the kinds of 'tensions' teachers encounter: *Have you ever had the experience, in the middle of a lesson, of suddenly wishing you were not there? Like a wave washing over you, you realise that things are not going well.*; Griffin [1989:13] writes about the difference between *being well prepared for a lesson and being prepared to act in that lesson*. Griffin also writes about tensions, about being prepared to act in the moment, and offers teaching strategies for use in the moment.

Mathematics from 5 to 16 (HMI), which built upon the Cockcroft report, was the most significant, authoritative publication to guide and support my developing pedagogy of teaching and learning; it examined a multitude of issues which were becoming central to my practice. For example, the report discussed the aims of teaching mathematics in holistic ways: *Mathematics is not an arbitrary collection of disconnected items*; of the *Awareness of the fascination of mathematics*; and, *Imagination, initiative and flexibility of mind in mathematics*.. [ibid., p.4].

³³ The journal of the Association of Teachers of Mathematics.

The report also endorsed problem solving as a way of motivating pupils: ... *a classroom where a range of activities is taking place and in which pupils express interests and ask questions can also provide on-the-spot problems. Teachers need to exploit these situations because there is a greater motivation to solve problems which have been posed by the pupils themselves.* [ibid., p.41]

Mathematics Non-Statutory Guidance [1989] served to confirm my beliefs, and in particular advocated integrating process skills³⁴ with content skills: *It is through engaging in these (using, communicating and developing ideas of argument and proof) that pupils will encounter the real power of mathematics. They are at the heart of mathematics, and should underpin pupils' work across all the areas of mathematics.*³⁵ NCC put further weight behind the importance of this issue: *The National Curriculum requires all schools...to develop a teaching and learning approach in which the uses and applications of mathematics permeate and influence all work in mathematics*³⁶.

These were the main sources which influenced my practice and supported my developing pedagogy.

Reading as researcher-self: June 1991-June 1996

At the beginning of my period of research I engaged with writers such as Dewey, Liebschner, Sotto and Shor. Each of these writers strengthened my values about teaching and learning, particularly my responsibility as a teacher for providing learners with active experiences to guide their learning. Dewey [1916:38] questioned passive approaches to learning: *Why is it, that in spite of the fact that teaching by pouring in, learning by passive absorption, are universally condemned, that they are still so entrenched in practice?*

Sotto [1994:22] wrote about the importance of doing in order to learn: *If you want to learn how to ride a bicycle, you have to ride a bicycle. If you want to learn how to bake a cake, kiss a girl, understand thermodynamics, or kiss a boy, you have to do those things. Explanations from somebody who already knows*

³⁴ Attainment Targets 1 & 9: Using and applying mathematics.

³⁵ NSG para 1.4 section D.

³⁶ Section 3.0 IMPLICATIONS FOR PLANNING AND CLASSROOM PRACTICE.

can help. But no matter how good the explanation, the best way to learn is when we are actively engaged. These student-centred methodologies confirmed my ideas about teaching and learning, and reinforced the importance I place on problem-posing.

Furthermore, my approach to writing this thesis mirrored that used by Sotto; he writes anecdotally and draws conclusions by analysing experiences. He also explores the ways people learn through questioning: *It is usually the teacher who asks most of the questions in a lesson, and we tend to take such a state of affairs for granted. However, it might be an idea to examine that custom for a moment. After all, the answers we understand and remember best of all tend to be the answers to the questions we have asked ourselves.* [ibid., p.179].

Shor guided me to towards learning as an empowerment, drawing strands together, such as problem posing and reflection: *By starting from the students' situation, problem-posing increases their ability to participate, because they can begin critical reflection in their own context and their own words.* [1992:45]. He writes about democracy in teaching and learning, and deals with critical thought and dialogue; describing Mathematics as: *one of the most abstract and specialized academic subjects, so it is a good place for problem-posing to show critical, student-centred approaches.* [ibid., p.76]. Having my ideas confirmed aided my writing and my ability to be explicit.

I was intrigued by the perspectives Dewey and Liebschner offered on educational issues, particularly the progressive, traditionalist dichotomy, which they describe in existence more than fifty and one hundred and fifty years ago respectively. For example, on the issue of *freedom* Dewey [1938:22] writes: *...an educational philosophy which professes to be based on the idea of freedom may become as dogmatic as ever was the traditional education which it reacted against.*

Liebschner writes: *Freedom could never be bestowed upon people, including children, but has to be worked for, thought about, defined, sensed, discovered and appreciated...*[1992:140].

Such debates continue as a focus of attention today.

Liebschner's discussion of the National Froebel Foundation report to the Plowden committee [*ibid.*, pp. 153-5], led me to Plowden in general and, in particular, to paragraph 505 which I quote in my introduction. This report formed a 'bridge' between the 1960's and the 1990's, and offered me a historical perspective about how many of today's issues in education were debated thirty years ago. Plowden offered a balance between issues of *practice* and *discovery*: *Practice is necessary to fix a concept once it has been understood, therefore practice should follow, and not precede, discovery.* [*ibid.*, para. 654].

Plowden also provided important perspectives on my opposition to setting: *There is also much evidence that streaming serves as a means of social selection.* [*ibid.*, para. 815]; *Selection will inevitably be inaccurate...* [*ibid.*, para. 816].

The Black Papers subsequently produced a backlash against unstreaming: *The movement for unstreaming has in recent years met with strong opposition from traditionalist writers on education...The Black Papers, indeed, appeared to link unstreaming with comprehensive education, student unrest and the permissive society as all leading to anarchy, or egalitarianism run wild.* Simon [1970:142].

Wilcox [1975:118] provided support for my rationale for not setting: *The work must interest and stimulate the children's imagination and lead to a pleasure in mathematical activity. Particularly important is the need for the work to be such that it maximises the child's chance of success. On the other hand the work must develop Mathematical ideas and give a view of the internal relationships and structures of Mathematics.*

Three reports, published during this period, which I drew upon, made reference to pupils' classroom experiences:

Dearing [1993:43]: *The aim must be to equip all young people - and notably those who are currently achieving little success in the core subjects - with the knowledge and skills they need to maximise their potential.*

SCAA [1993:27]: *...dividing pupils into rigid sets or groups according to their previous results is likely to lead to considerable inequities.* ³⁷; and

³⁷ Summary Report.

Ofsted [1994:23]:in the first year of secondary schools there are several problems. Much of the work fails to stimulate interest...work is not differentiated to suit different abilities; 40% of the work in mathematics is too easy especially for more able pupils; there is too much repetition for able pupils and too little for those who are less able.

Each of these reports validated my work and supported my beliefs in the importance of offering pupils opportunities to engage with mathematics in stimulating, problem-solving ways.

A source offering perspectives, on general educational issues, was FORUM a journal aimed at *promoting 3-19 comprehensive education*. This journal forged links and combined issues, a process at the heart of research. For example, a quote from Gillard [1992:92] served to connect together issues raised earlier by Plowden and later by Donaldson: *I make no apology for believing that most of what Plowden had to say about children and their primary schools was - and still is - absolutely right. I go along with Margaret Donaldson that what we should be doing is extending good primary practice into the lower secondary years...*

Finally, Moon & Mayes [1994]: *Teaching and Learning in the Secondary School*, offered me a breadth of perspectives on a multitude of issues. For example Chapter 3: *A question of ability?*³⁸ provides a brief history about streaming and mixed-ability and concludes: *the improvement of practice must be the recognition that no child's potential is fixed.* [ibid., p36]. Chapter 29: *The entitlement curriculum*³⁹, is reproduced from a report of an enquiry published in 1977. The concluding remark reads: *The aim to develop curiosity, creativity and independent thought will not be achieved by teaching which relies excessively on instruction and didactic methods. Teachers have shown they need to adapt various teaching styles; they are at times listeners, at times partners, at times assessors; they need to question, cajole, encourage and guide and to know when, how, and when not to intervene. Teachers must have the means to enable the entitlement curriculum to be achieved.* [ibid., p.240]

³⁸ Bourne and Moon [1994].

³⁹ HMI [1977].

Reading as researcher-self: June 1996-September 1996

This proved to be my most prolific period for reading. I made many more connections between authors and with my writing. Some writers challenged me to consider my pedagogy in greater depth, none more so than Freire [1972:53]: *The (problem-posing) teacher is no longer merely the-one-who-teaches, but one who is himself taught in dialogue with the students, who in their turn while being taught also teach. They become jointly responsible for a process in which all grow. In this process, arguments based on 'authority' are no longer valid; in order to function, authority must be 'on the side of' freedom, not 'against' it.*

I read Educational Studies in Mathematics and For the Learning of Mathematics, both from 1987; consequently I engaged more profoundly with issues of constructivism and learner-centredness and recognised I held similar values: *Problem-centred learning is an instructional strategy based on constructivism.* von Glasersfeld [1987:529].

I concurred wholeheartedly with Lochhead[1992:535]: *A constructivist teacher must hold fast to two essential attitudes. The first is the belief that all people can learn, that incompetence is mostly a construct of the imagination.. The second is the willingness to provide students with the opportunities to devise their own learning unimpeded by teachers' helpful suggestions.*

Wheatley's writing about being reflective deepened my conviction about the value of this as a powerful learning tool: *Persons who reflect have greater control over their thinking and can decide which of several paths to take, rather than simply being in the action. It is not enough for students to complete tasks; we must encourage students to reflect on their activity.* [1992:535]. I subsequently made stronger connections between reflective learning and spective awareness.

Other educationalists, such as Gattegno, Freudenthal and Schoenfeld helped me clarify and articulate issues about teaching and learning. I was constantly struck by the way they exposed ideas and described complex issues in ways which made them appear 'obvious', and 'transparent'.

For example:

All I must do is to present them (the pupils) with a situation so elementary that they all master it from the outset, and so fertile that they will all find a great deal to get out of it. Gattegno [1963:63];

Traditionally, mathematics is taught as a ready made subject. Students are given definitions, rules and algorithms, according to which they are expected to proceed. Only a small minority learn mathematics in this way.

Freudenthal [1991:48]; and

When mathematics is taught as dry, disembodied, knowledge to be received, it is learned (and forgotten or not used) in that way. Schoenfeld [1994:60].

Concluding remarks

Future research opportunities may emerge as a consequence of learning from the methodology I have adopted during my research. No single research method is infallible and the findings of research are best validated through triangulation of methods. Consequently, there is potential for this case study to be further tested out by:

- other people comparing my work with their own by adopting a similar, reflective practitioner, approach to their research;
- an outsider studying my practice;
- myself studying other peoples' practices using the skills and insights I have gained through the methods I have used to research my practice.

Chapter 3

Classroom Atmosphere

Three main symbols of authority have shared between them the attention of the world: the slave-driver's whip, the shepherd's crook and the conductor's baton. A reasonable man should make up his mind which of the three he prefers: which he will submit to when it is his turn to submit and wield when the time comes for him to rule. (Mary Boole) Tahta [1972:12].

Classroom atmosphere is a multi-faceted collection of elements which to define in two or three sentences would be impractical. However I find Boole's classification of three types of authority useful in trying to characterise the basis of my classroom atmosphere. The atmosphere I seek to create is based in an environment constructed on values of equality², within which I can teach, and learners can most effectively learn mathematics.

Outline

In this chapter my central aim is to describe how my values have developed, to foster a supportive classroom atmosphere and the consequent effect upon childrens' learning. Boole causes me to reflect upon my mode of teaching at particular times, and whilst I cannot consistently be the kind of teacher who utilises the conductor's baton, it is from this perspective I set out. She provides me with a challenge and an important image of creating a conducive climate, an atmosphere where pupils are encouraged to engage positively in their learning of mathematics.

In the first three sections I develop issues of identifying, describing, and creating and gauging the elements which define classroom atmosphere. By analysing events from my, and other teachers', classrooms I construct criteria for gauging effective classroom atmosphere.

The elements, which shape and frame my classroom atmosphere, are:

- display work,
- happiness and challenge,
- confidence,
- responsibility and ownership: subordinating teaching to learning,
- co-operation and competition,
- trust, and
- caring.

Each of these are sub-headings in this chapter.

Identifying classroom atmosphere

Identifying classroom atmosphere may be considered a nebulous act. In order, therefore, to demonstrate the value of identifying elements which create a supportive classroom atmosphere, I have drawn upon extracts from lesson observation notes written by Judy Watson, Head of English and mentor for initial teacher trainees at a School in Weston-super-Mare¹.

Notes on Student A teaching a Y8 class:

The atmosphere was relaxed and pupils were obviously enjoying watching the performance. Your manner and approach with the class were absolutely appropriate. You were very positive, calm and receptive.

and,

Notes on Student B teaching a Y10 class:

I was impressed to find a very relaxed and purposeful atmosphere... when you were addressing the whole class, you stopped twice to wait for full attention which was an important and necessary strategy. You moved around the room supporting and guiding and I was impressed by the good relationships you have established in such a short time.students were working in

¹ I was fortunate to have access to these notes as a consequence of my role as external examiner for a PGCE course. The extracts have been reproduced with the kind permission of Judy Watson.

pairs to brainstorm and plan an essay title. The atmosphere was very purposeful and some excellent work was being done... I heard contributions from all but one of the students. Your feedback was warm and encouraging and again your questioning challenging. You gave plenty of praise and finished the lesson positively - students left the room knowing what they would be doing next lesson and feeling confident about their performance. This was an excellent lesson to observe and you have done really well.

Judy is blind. Many of her perceptions may be the same a sighted person would recognise; she may also be sensitive to additional features which a sighted person may overlook or think unimportant. The implications are that whilst atmosphere may, in part, be identified by visual observations there exist other features which transcends this, for example the ways teachers and pupils talk and respond to each other. Her notes suggest atmosphere is a set of observed sensory perceptions; this helps me define how 'atmosphere' can be recognised. In whichever way she perceived the signs of atmosphere being *very purposeful*, her notes demonstrate the importance of atmosphere as a basis for an effective and supportive learning environment.

Defining and seeking a positive classroom atmosphere

I define a positive classroom atmosphere as a lesson where opportunities exist for pupils to engage determinedly with learning.

I have become increasingly conscious of the atmosphere in my classroom; as a teacher trainer I am strongly influenced by the atmosphere I sense in students' classrooms. An effective atmosphere embraces meaningful relationships between teacher and pupils, it creates and builds upon positive interactions. These lay the foundations for 'control' and 'good discipline'. How this is achieved, and how those qualities which define an effective atmosphere are recognised, characterised and described, I attend to below.

To create a positive atmosphere, there are some essential 'basics' related to lesson planning. These are to construct accessible² and interesting tasks². What happens once a lesson has begun is not predictable nor can events be legislated for. As a lesson unfolds a classroom atmosphere is created; the lesson outcomes will depend upon the individual style of the teacher and the nature of interactions which occur. Over time the types of interactions become a distinctive feature of the teacher's style.

Another 'basic', related to classroom atmosphere, is learning pupils' names. Below I describe a method which is idiosyncratic to my style and is an approach I often use with a new class.

It is the beginning of the lesson and I am standing close to the door with a pile of new exercise books. As the first pupil enters the room I ask their name; upon repeating their name I hand the pupil a book. The next pupil enters and I repeat as before but this time I also repeat the first pupil's name. This continues and I sense the pupils' expectations rise as more names are added to the list. The pupils are amazed as this onslaught of naming continues; their attention is focused on whether I can complete the task without error. *"Will he be able to remember them all?"* is an unspoken question which I am confident is in some pupils' minds. Once everybody is in the room, and I have repeated the list many times, I ask them to sit in a different seat. The pupils do this with minimum fuss and once again I attempt to say each pupil's name.

Such a beginning to a new term sets the scene for further meaningful interactions with a group; I am demonstrating to pupils the following:

- getting to know each other is important;
- I work hard at learning something (their names);
- surprises occur in my lessons;
- learning by constant repetition (in moderation) can be enjoyable.

² I develop this in the 'Curriculum Development' chapter p. 105.

The anecdote typifies my way of working; it may sound hectic, yet my experience has bred an inner-confidence. I would not suggest another teacher adopt this strategy; it is part of my style, although the approach may be adapted. Learning names in this way is a small aspect of my practice; a beginning from which a positive atmosphere, based upon knowing each other, is seeded.

Gauging an effective classroom atmosphere

In lesson observation notes for PGCE students I frequently comment upon classroom atmosphere. I also analyse elements which form the atmosphere in my classroom and gauge the 'success' of lessons against the occurrence of certain types of events. A key effectiveness indicator is whether I would have been content for my PGCE students to have been 'flies on the wall', observing to see whether I construct the kind of atmosphere which I advocate in support of effective learning. My list is based upon the types of interactions I have with pupils, the ways they respond to me, and the types of tasks I offer:

- as pupils enter the room there are pleasant exchanges between us;
- during the lesson I find opportunities to celebrate pupils' achievements and show I care about how they work;
- in an introductory lesson^s to a module^s, I find an interesting starting point^s from which all the pupils can potentially access the intended concepts;
- the work provides opportunities for pupils to work at different paces and develop tasks commensurate with their aspirations, aptitudes and interest;
- for a continuation lesson^s, the majority of pupils can organise themselves and begin work without needing me to signal the 'lesson has begun';
- various pupils choose to talk to me about their work, indicating a keenness to make progress and take responsibility for the work they do;
- the amount and type of chatter does not cause me to be an overbearing, authority figure;
- packing away is achieved in a calm fashion;
- at the end of the lesson, on leaving the room, there are pleasant exchanges between myself and several pupils, as we bid each other farewell.

To illustrate the dynamics of my classroom, I offer the following anecdote from July 1994 with a Y8 group³. I describe various behaviours and interactions which occurred and show my responses to particular incidents.

The context of the lesson

- The group were extremely demanding, and were recognised throughout the school as 'difficult'.
- There were several pupils with learning and behavioural difficulties.
- The range of motivations, responses and workrates was as variable as the web of relationships within the group; this included certain children behaving antagonistically towards each other.
- The lesson fell at the end of a Tuesday afternoon and their previous lesson was P.E. Consequently pupils tended to arrive over a ten minute period in a fairly high, thirsty state with many social issues unresolved.
- During the last period on Tuesday afternoons I was conscious of the need to be well planned and alert to the demands of many of the pupils' behaviour.

The intended mathematical content of the lesson

The module was based upon visualising, classifying and representing 3-d shapes using multi-link cubes. I had introduced the work in the previous lesson with a task I call Telephone Cubes⁵; this involved pupils sitting back to back with one pupil describing to another, as over a telephone, the shape s/he had made.

My reasons for starting the module in this way was to offer an interesting introductory focal point, and to provide pupils with an opportunity to initially play⁴ with the equipment. The main task was to make 3-d shapes from 4 cubes, produce iso-metric drawings of them, and try to find a system to show they had found them all. An extension⁶ task, which I describe to small groups, as pupils complete the 4 cube problem, is to find all the possible cuboids from 12 cubes then work out the dimensions and surface area for each cuboid they make.

³ Because I have used this anecdote to describe a wide range of issues about my practice, I have not included it in the list in appendix 1a or the table in appendix 1c.

⁴ Whenever I introduced different practical equipment to a group I purposely allowed a short time at the beginning of the lesson for pupils to acquaint themselves with the equipment.

The anecdote

I take up the anecdote at the beginning of the second lesson of the module.

The first person into the room is Rakshe who frequently demonstrated behavioural difficulties; I had already decided I was not going to tolerate her moodiness today. She came in waving a small piece of yellow paper which she wanted me to sign so that she could go to the medical room. "No" was my instant thought.

During the course of a lesson teachers have to make many on-the-spot decisions and judgements. Being aware of the possible consequences of how I respond is consistent with the 'Noticing' paradigm⁵. Recognising I will not always make the most appropriate judgements in response to a situation is important, however, this does not release me from my responsibility of noticing in-the-moment.

As a result of her anti-social behaviour in the previous lesson, I decided she would work in the quietness of the Maths store.

Whilst in general I believe in the principle of inclusivity⁶, of including pupils rather than segregating and excluding them, I must also accept there will be occasions when this may not always be achievable. Furthermore I have to balance issues of inclusivity for one pupil against other pupils' rights to a beneficial learning environment. This was such an occasion; over numerous lessons I had deployed all my known strategies yet Rakshe continued to behave antagonistically to teachers and pupils alike. However much I regretted having to temporarily exclude her, I nevertheless had responsibilities towards the rest of the group. I can only justify such actions by accepting my limitations and recognising that occasionally some pupils demand more than I can offer.

As this event is taking place, the rest of group are still entering the room and I contemplate previous conversations I have had with their P.E. teachers, about the difficulties of pupils arriving to this lesson in "dribs and drabs", and ways this might be avoided.

⁵ See Methodology chapter pp. 29-30.

⁶ I develop issues of inclusivity in greater depth in the Un-setted chapter pp. 172-175.

A group of six pupils bid to sit around a space intended for four. Past experience tells me that Sue, Peter, Liz, Lin Lee, Hannah and Clare do not always work well together, particularly on a Tuesday afternoon. I ask them to split themselves up, which they do with minimum fuss. However, Lin Lee's command of English can be remarkably poor when convenient, and she throws a spare cube across my desk aiming for the tray (it misses). I spend a few moments discussing this with her.

In the previous lesson, throwing cubes into the box had become a 'good' packing away 'game', so I had already decided 'we' would work on this aspect of the lesson ending. I had not however envisaged needing to work on this at the beginning of the lesson.

Meanwhile Clare, who had been absent from the previous lesson, and who often needs a lot of my time and attention, is standing next to me 'limpet-like' waiting for my help.

Whilst I know she requires input, I also need to attend to the rest of the class as they continue to enter the room. Past experience informs me that for this lesson, I need to ensure the whole class are settled before attending to individual needs. Once the class are settled I can provide the attention different pupils require

Lyndsey, an MLD-statemented student is shouting some abuse at John who, she claims, called her "Monkey eyes", in the corridor after P.E. I quietly calm her down and ask John, who according to his mother at a recent parents' evening "loves his maths lessons", to move away from Iain, Jane and Ahmed; whom I suspect are keen to give succour to John's behaviour. In a very controlled manner I ask Lyndsey to write about why she is shouting. Clare, meanwhile, is still standing next to me and I forget to use the "*Mantle of the Expert*" strategy⁷.

Despite sometimes feeling like a performer keeping a number of plates spinning on the end of sticks, I am also aware of the need to demonstrate control, patience

⁷ See Teaching and Learning chapter pp. 145-147.

and an inner and outer calmness in my behaviour. Past experience tells me that raising my voice does not have a long lasting effect upon the group; indeed this only adds to the sense of 'drama' pervading the room. I therefore seek to respond carefully to each incident in a calm and patient way

Peace! Almost ten minutes into the lesson, after what seems to be its usual hectic start, the group are working very quietly and with a deal of purpose. I feel to be conducting an orchestra. I continue to be very calm and I quietly ask Ahmed and Jane to settle to their work. Liz⁸ has done a lot of homework and is keen to show me. I remember Raksha is still in the store, and having previously claimed not to know what she was meant to be doing, I ask Liz to help her. She does so happily, and returns a few minutes later.

Twenty minutes into the lesson and I explain an extension task to ten pupils. This is to work out the dimensions and surface areas of the cuboids they have previously made and drawn.

This strategy, of collecting together a number of pupils who, I assess, are ready for an extension task, is one I regularly use. Despite the earlier focus on behavioural aspects and awkward social interactions between pupils, a deal of positive mathematical activity has taken place. The extension task has been well received and pupils have settled to the new task, one aspect being to find a complete set of possible solutions.

Liam has chosen to work quietly on his own; I am conscious he is putting a deal of thought into his work. Iain, who often displays poor work practices, is deeply involved and is keen to find all the possible solutions; he keeps coming to me to show how he is getting on. He is displaying more motivation and interest than I have seen from him throughout the year; by the end of the lesson he has accurately worked out all the possible

⁸ This pupil had some of her work published on the front cover of MT 148.

cuboids and their surface areas from twelve to two cubes. I am very pleasantly surprised.

Iain's response was well beyond his previous standard. With regard to not setting there is a centrally important principle that all pupils are provided with opportunities to work at a pace and a level commensurate with their interest and aptitude. This is significantly different to placing pupils in a set according to previous work habits, or based upon the result of a test. Not setting demands that I work in-the-moment, using my perception of pupils' present and different needs. Providing pupils with an equality of opportunity, to raise their usual standard to a higher level, is synonymous with the principle of not setting.

The initial bustle at the beginning of the lesson has evaporated. The pupils' general work rate, especially for this 70 minute lesson on a Tuesday afternoon, and particularly in the last two weeks of term, has been most pleasing.

I make a point of praising the work they have done and we pack away in a careful manner. As we are doing so Liz asks me if she can take some cubes in order to carry on the work at home, three other pupils make the same request.

I was happy to agree to these requests; whilst letting pupils take equipment home may appear risky, I am clear about my decision to respond positively. I wish to take opportunities, whenever they emerge, to offer pupils trust⁹; were I to decline this request I would undermine the trust I seek to build.

Chairs up - time to go - now gone and there is Liam waiting to tell me there is less than a month to go before the beginning of the 1994/95 football season!

Liam is one of the stronger mathematicians in the group and whilst he only occasionally volunteers to discuss his work with me he often stays behind at the end of a lesson in order to strike up a conversation about Shrewsbury Town or to test me with questions like "Who will win the World Cup?"

⁹ See 'Trust' in this chapter pp. 68-72.

Such conversations are important. I choose to encourage them in order to develop meaningful relationships upon which pupils feel at ease whilst learning mathematics. I also wish to demonstrate that I share certain, common interests and discussing them, at an appropriate time, is an accepted part of my classroom atmosphere. Fostering such situations is linked to my craft as a teacher; I work hard on this aspect of my teaching.

A further strategy which I deploy, to build working relationships between myself, pupils and their mathematics, concerns the work displayed in the classroom. I develop this in the following section.

Display work

As a teacher trainer I have been in many classrooms and my attention is drawn by the amount of pupil work displayed on the walls, doors, windows and ceilings. In some classrooms I begin to sense I am in a place with an atmosphere, one which elevates pupils' learning through a public acknowledgement of various achievements. Other rooms have bare walls, or age-old displays which are torn and peeling; here I sense an atmosphere which, seemingly, doesn't value the importance of the working environment. In other classrooms I see display work with a focus on 'near perfection', and I ask myself if this implies only certain types of work are displayed.

In my classroom I display work which is neither 'perfect' nor, sometimes even complete. I set out to fill every possible space, and only when this has been achieved do I replace earlier work with newer display work. I also display work I do. For example, in a lesson which began with paper folding⁸ providing opportunities for pupils to produce tessellations designs, I also produced a design which I displayed alongside the pupils' work.

Display work can arise from various situations. On one occasion, at the end of a module of work on factors with a Y8 class, I gave each pupil a brightly coloured piece of A4 paper and, for homework, asked them to produce a small poster to describe an aspect of the work they had done. The results were pleasantly

surprising and some pupils had clearly spent much time producing intricate designs. For example, one pupil had drawn a cartoon character who explained how to work out the prime factors of a number. These designs subsequently formed a display to illustrate the work done within the factors module.

I also used display work as a focus for discussion.

I am working with a Y10 class on quadratic graphs. I give each pupil a copy of the graph of $y = x^2$ and, on the same axes, I ask them to draw a second graph such as:

$$y = 2x^2,$$

$$y = x^2 + 2,$$

$$y = -x^2.$$

As pupils complete their graphs I instantly display¹⁰ their work. Next we gather around the display to discuss and analyse the effect of changing the coefficient of x^2 and adding a constant, we discuss similarities and differences of the graphs.

In this way the display was used as a teaching and learning aid¹¹.

Display work can also be used as a strategy for:

- providing variety of experience to utilise different pupils' skills;
- devising imaginative and creative approaches to mathematics;
- enjoyment;
- group work;
- motivating pupils from other classes who use the room;
- encouraging decision-making about scale and how best to display ideas;
- developing pupil responsibility and ownership¹²;
- sifting out, summarising and conveying what has been learnt¹³.

¹⁰ I found Copydex glue an invaluable resource for speedily producing instant displays.

¹¹ In the same way, functions based upon the sine curve can be explored.

¹² I develop issues of responsibility and ownership later in this chapter.

¹³ In March 1997 I asked a group of 1st year undergraduate students at the University College of St Martin to produce a poster to summarise their Mathematical Modelling assignment. Many of the resulting posters were of an outstanding quality.

Berry and Houston [1995: pp. 21-27] have carried out some work in this area and their research supports this approach of learners producing posters:

... research in other disciplines suggests that poster sessions for students: are an excellent alternative medium for developing communication skills,...promote a positive attitude in students.

and

Posters require succinctness and so students have to select the most important things in their work. [ibid., p. 23].

When colleagues, parents or visitors enter my classroom I intend that they sense a positive atmosphere; an environment which promotes and celebrates¹⁴ pupils' learning. Pupils are affected by their working environment; it is important, therefore, to pay attention to it.

Happiness and challenge

Mathematics must be an experience from which pupils derive pleasure and enjoyment. DES [1985:7].

In the earlier section on gauging atmosphere, I referred to "*pleasant*" interactions during lessons. Constructing an atmosphere where pupils find happiness is central to my practice; however this must be interwoven with appropriate mathematical challenges; devising problem solving approaches¹⁵ is, therefore, central to my pedagogy of learning mathematics. Here I consider how happiness in learning must be juxtaposed with appropriately challenging tasks, so that learning mathematics has its own intrinsic value.

I illustrate this with an anecdote from a Y9, which, incidentally, is the same class eight months later from the date of the first anecdote. The content of the lesson was constructing and solving linear equations based upon the Pyramids⁸ idea which I had first encountered in the South Nottinghamshire Project⁸.

¹⁴ I first engaged with the notion of celebrating pupils' achievements at an ATM-SEG GCSE weekend when Jude Stratton (formerly Head of Mathematics at Cheney School, Oxford), used this phrase to describe an aspect of assessment.

¹⁵ A typical problem would be to find all the triangles on a 9-pin geoboard.

A pupil's pleasure of learning to use algebra

The event occurred during a discussion with three pupils about how to create an equation in order to solve a problem, and how this was more powerful than the trial and improvement method they had previously used.

John, Craig and Peter show me their solutions to the Pyramid problems saying they have found another method, other than by guessing, for working out answers. They have yet to formalise their thinking algebraically and I decide this is an opportune moment to help them do this. I begin by asking how they have solved the problem, then describe how to build up the equation from the situation, solve it, then check the result.

Throughout this interaction I assessed, in-the-moment, what I believed they knew and may be able to understand further. They were clearly pleased at being able to find a method, involving algebra, to solve the problems. What occurred next confirmed my belief of working to create an atmosphere based upon happiness, which is simultaneously challenging.

Suddenly, out of the blue John, exclaims "*REBEL*". I make no attempt to hold back my amusement at this sudden, public outburst of enthusiasm; there are smiles all round. I ask him what he means by the word rebel, to which he replies: "*Well you know, it's wicked*". We laugh again and the three of them return to their desk to try to construct equations for other problems.

Their pleasure at being able to recognise how algebra could be used to solve problems was clear. Setting this further challenge, to develop their abilities to work in a more abstract way, had been appropriate.

As they begin work on further problems I continue to smile and consider the situation of a Y9 pupil, who hasn't outwardly shown a strong interest in mathematics before, being able to show his joy of using algebra, and feel quite comfortable at expressing his pleasure in a public manner before his peers.

The method I offered was one which clearly interested John and his reaction had a positive effect on others in the group, giving validity to the nature of the work they were doing. Through an appropriate challenge John had been able to derive pleasure in learning mathematics.

Another important element was the use of a problem solving approach which enabled pupils to perceive the need for, and the value of, using algebra. This approach contrasts sharply with one where pupils are provided with equations to solve from an exercise devoid of context of the challenge of problem solving.

This incident, the manner in which it occurred, and my response, were all important threads which form the web of the atmosphere in support of pupils learning mathematics.

Personal pleasure of understanding vector spaces

A second event from 1985 describes a personal struggle and my happiness in gaining enlightenment at overcoming a challenge. The context was studying the Open University M203 course¹⁶:

I had been struggling to understand something about vector spaces and had gone through many emotions, which I am sure many learners feel; frustration, despair, giving in and, for some strange reason, waking up in the middle of the night and having a solution to a problem.

Later that day when I returned to my work with my new found understanding other concepts, which I had been struggling with fell into place, I made such a celebratory exhortation my partner thought I'd had an accident. I leapt downstairs and even though I knew she would not understand the concept of vector spaces, I still needed to explain my 'eureka' moment to her.

This describes a relatively momentous aspect of my learning. My pleasure at having solved a difficult problem was all the more significant because of the

¹⁶ Introduction to Pure Mathematics.

complexity of the challenge I had faced. I also recognise the joy of making smaller steps, such as learning a new chord on a guitar. This pleasure deepens when I play this chord with established chords and I enhance my ability to play a guitar. Being aware of one's learning, whatever the magnitude is important.

Happiness however, in the context of classrooms, has been criticised by some politicians in recent years, with some primary classes being described by Michael Fallon, then Education Minister as:

playgroups where there is much happiness and painting but very little learning. Sweetman [1993:9].

Dewey [1938:26] was more circumspect:

An experience may be immediately enjoyable and yet promote the formation of slack and careless attitudes.

To avoid the *formation of slack and careless attitudes*, appropriate challenges must be provided and a range of teaching strategies used. However, a challenge on its own is not sufficient to promote effective learning if the learner does not derive pleasure from the pursuit of solving the problem, or ownership of its solution; for instance, completing an exercise may be a challenge, yet a pupil may not learn any mathematics. Jaworski [1992:14], connects challenge with the management of learning and sensitivity to students:

Good management depends on sensitivity, if it is to succeed. Challenge cannot be taken up if it is inappropriate, or if strategies for handling it have not been created. Sensitivity alone might create happy situations, but challenge is required to enable mathematics to be done. Through good relationships which arise with students the teacher can gain access to their thinking. Through established ways of working the teacher can expect challenges to be taken up. By examining the response to challenge, the teacher can gain insight to levels of construal. This insight enhances knowledge of students and provides a basis for continued challenge.

Happiness and challenge are important in their own right as elements to support learning; when these elements are combined and pupils are empowered, as learners, to gain pleasure from rising to the challenge of solving a problem, then effective and potent learning occurs. By teaching mathematics, therefore, as a problem solving activity, happiness and challenge simultaneously co-exist, and an empowering classroom atmosphere can be achieved.

Confidence

Building confidence to support pupils' learning of mathematics, is a key element of classroom atmosphere which I place much importance on. To illustrate this I describe events from a lesson in a trigonometry module with a Y11 class from September 1991; in the previous lesson I had described the rotating arm scenario¹⁷, and followed this with questions about what happened to the co-ordinates at the end of the arm. The basic knowledge pupils needed was:

- a scale from 0 to 1 being divided into ten 0.1 divisions;
- angular measure as an amount of turn;
- the co-ordinate system in the positive quadrant.

I had previously worked on all these ideas so none of this knowledge was 'new'. For example all pupils had used co-ordinates in contexts such as functions, graphs and transformations. For some pupils, therefore, the starting point would be a reminder of previous concepts, whereas for others, the starting point would re-establish knowledge. I take up the anecdote at a point where I recognised that Suzie, who often struggles to concentrate for long periods, was displaying obvious disinterest.

Suzie had engaged really well in the first lesson, yet has switched off in this lesson. I am aware that Suzie can respond, in negative ways, in similar situations, so I choose not to act immediately, and instead decide to give her time and space, believing this is what she needs

¹⁷ See Ashworth, K. [1995].

Teachers make numerous decisions about when to intervene and when to stand back. The choices I make depend upon many factors; at my most effective I attend to pupils' needs in careful and considered ways. Building a pupil's confidence may sometimes involve stepping back and waiting.

Later in the lesson I decide it is appropriate to discuss with Suzie what she is struggling with. She says she doesn't understand the work so I deploy the strategy of asking her to restate what she had already done.

This approach, of asking pupils to reconstruct the work they have done, is one I frequently use; I call it the 3 d's strategy¹⁸.

Suzie explains she has completed the initial task, of working out the co-ordinates in the positive quadrant and can see certain connections in the way the co-ordinates increase and decrease. She has drawn a second diagram for angles between 90° to 180° and I ask her what she can't do; she tells me negative readings are going to be required.

Recognising the need to use negative values for the horizontal readings was the crux of the problem. Why did Suzie say she did not understand when she showed much insight into the problem? I attribute this to her lack of confidence, not in what she had done and understood, but what she believed she wasn't going to understand. This lack of confidence, of moving from the known into a partially unknown area of mathematics, is different to pupils not presently understanding. Papert [1980:42] describes lack of confidence as "Mathphobia":

If people believe firmly enough that they cannot do math, they will usually succeed in preventing themselves from doing whatever they recognize as math. The consequences of such self-sabotage is personal failure, and each failure reinforces the original belief. And such beliefs may be most insidious when held not only by individuals, but by our entire culture.

¹⁸ See 'Teaching and Learning' chapter pp. 143-144.

This approach, to help Suzie confirm what she already understood, then build her confidence to believe herself capable of taking further steps is a key strategy.

Developing pupils' confidence requires helping them:

- build on previous experiences to gain further knowledge;
- recognise what they have already achieved; and
- believe further steps are possible and are within their capability.

Presenting children with interesting and accessible tasks is vital for building confidence and fostering a desire to learn. I link access to entitlement as follows: For all pupils to have equal access to mathematical concepts, as defined by National Curriculum orders, I have a responsibility to provide them with appropriate, common starting points from which they can construct meaning. These meanings are based upon pupils gaining confidence to move from the known to the unknown and to control and develop tasks. My role is to guide pupils towards explicit recognition of the mathematics implicit within the tasks. Pupils must know what they have learnt as a result of carrying out a task. Building confidence, when certain concepts might ordinarily¹⁹ be considered beyond some pupils' capabilities, is particularly important when working with pupils in un-setted groups²⁰.

Responsibility and ownership: subordinating teaching to learning

I don't think that children are given enough power in school to work efficiently, carefully and with a sense of ownership of the work. Heathcote, D. [1991:VTR10].

Finding ways to help pupils to take responsibility for their learning is one of the most important elements in my teaching. Responsibility grows from ownership; seeking ways of enabling pupils to own their knowledge, and take an active

¹⁹ I use the word 'ordinarily' within a wider UK context, where the majority of pupils are placed in sets and as a consequence are provided with different curricular. I develop this point in the 'Un-setted' chapter pp. 170-175.

²⁰ An illustration of this appears in the BBC 'Teaching Today' programme (January 1995)

responsibility for their learning underpins the construction of a positive classroom atmosphere.

The following anecdote from May 1990 describes events with a Y7 group when I was twenty minutes late to the lesson. It exemplifies the importance of ownership and pupil responsibility.

I am expecting visitors from another school who wish to observe ways of teaching mathematics without using SMP texts. They also want to observe mixed-ability teaching in action.

The visiting teachers relied heavily upon the SMP 11-16 scheme and the HoD had requested this visit as a result of attending InSET some months earlier in Wolverhampton.

Today is the final assembly for Y11 pupils before beginning GCSE study leave; I am a Y11 form tutor and wish to be present. Because I expect the assembly to run over into the first lesson, I use part of registration time to visit my Y7 class to inform them of this and tell them to expect some visitors to arrive.

Trust²¹ also plays a central role in helping pupils gain responsibility. Because I expected to be late to a lesson, it was important I could trust the group to take responsibility in this situation.

My Y7 class know where everything is i.e. felt pens, glue sticks, sugar paper etc., so they can continue with their posters on the 'Snook' problems.

The class were continuing a poster-making task which they had begun in the previous lesson so I had to ensure they could access the equipment.

The assembly finishes 20 minutes late and I duly arrive in the classroom. There is a hive of activity; pupils are busily engaged in their tasks, and some are taking the opportunity to discuss

²¹ See 'Trust' later in this chapter pp. 68-72.

their work with the visitors. The atmosphere is busy, yet calm; my appearance is greeted with friendly comments from several pupils.

In a small, yet significant way I had been able to subordinate²² my teaching to the pupils' learning:

A radical transformation occurs in the classroom when one knows how to subordinate teaching to learning. It enables us to expect very unusual results from the students - for example that all students will perform very well, very early and on a much wider area than before...

...it is so different from what has been going on for so long that it requires a true conversion from the educator, so he no longer neglects to consider the most important component of education, the learner himself.

The consequence of including the learner - which means that the classroom process of learning becomes one of self-education, the only real kind - is that teaching techniques and materials must be recast. Gattegno [1971: ii]

Throughout the remainder of the day the visitors observed several different mathematics classes; we met at the end of the day for final discussions. To my surprise they focused on events at the beginning of the first lesson describing their amazement at how children, without a teacher in the room, were able to organise themselves, get the appropriate equipment and begin their work. They compared this situation with their own, describing how it would be impossible in their classrooms to offer pupils such freedoms; how some pupils would have abused the use of equipment and others may have stolen it. This interested me because what had occurred, matched the expectations which the class and I had constructed during the year. These expectations were based upon a classroom atmosphere based upon trust, pupil responsibility and ownership.

²² I define the subordination of my teaching to pupils' learning as a shift of focus away from my teaching needs to pupils' learning needs

Co-operation and competition

Cooperative learning foreshadows cooperation in adult life and professions. Freudenthal [1991:180].

I emphasise the value of co-operation as an important way in which pupils learn. I organise some lessons where pupils compete with one another; I describe an example of this below. More generally I use and define competition as:

- encouraging pupils to compete with the problems I pose;
- pupils competing against their previous best efforts.

I use the following outline lesson plan with many different groups; changes to the detail and decisions about extension tasks²³ are made according to the class I am planning to teach.

Four-in-a-line²⁴

This task requires pegs and pegboards. Each board can be marked out, using chalk, with horizontal and vertical axes and numbered accordingly. Games are played with two people, each having ten or so pegs of their own colour.

The aim is to try to get 4 pegs in a line, either vertically, horizontally or diagonally without an opponent's peg interrupting a line of four.

Lines at angles other than 45° are acceptable.

Pupils take turns and once an uninterrupted line of four pegs has been made, all other pegs are removed. The winning line is recorded as a set of ordered pairs.

Possible extension tasks are:

- *predict the two points at either end of each winning line;*
- *find patterns in the co-ordinates of the winning points;*
- *seek a connection between the horizontal and vertical numbers;*
- *extend the vocabulary of intercept and gradient;*
- *look for a strategy for winning the game;*
- *play the game, on paper, with someone at home*²⁵.

²³ An extension for some pupils is to rotate the board, about a centre point, through 90°, 180°, etc., then work out the equations of the resulting lines.

²⁴ This task is fully described in 'In Our Classrooms', ATM pp. 9-10.

²⁵ I found a homework task, such as this, can be useful for involving parents.

My usual strategy is to ask two volunteers to each play a game with me for demonstration purposes. Having established the procedure I ask pupils to play up to a dozen games and record the results of their winning sets of points both in their books and also with large felt pens on strips of sugar paper²⁶. Further tasks for pupils are described above. At the end of the first lesson I collect together all the sets of points which produce parallel graphs, and before the next lesson I display these on separate, larger sheets of sugar paper. Each poster is a family of graphs²⁷ and I display these in the classroom for the following lesson. The posters become the main focus of the lesson; the task is for pupils to draw each family of graphs on a separate pair of axes, work out the names of each graph and to see how the graphs in each family are connected.

During the first lesson pupils compete with each other; in subsequent lessons they co-operate and develop their understanding of co-ordinates, negative numbers, equations, functions and graphs. Understanding is achieved by active engagement in an equipment-based task and through pupils being encouraged to discuss with each other the work they are doing. A situation which began with competition shifted to one of co-operation.

To cooperate, to work together, to give up some of one's individual behaviour in favour of collective behaviours, to collaborate in the pursuit of knowledge and in search for common good, are essential goals of the school.

D'Ambrosio, U. [1990:22].

Competition and co-operation can be co-existent ways of working. To argue for one and not the other, is to polarise debate. Such polarisation serves to miss the point of how different perspectives can sit comfortably together and, therefore fail to engage with the central issues of how children learn most effectively²⁷.

²⁶ These pieces of sugar paper are approximately 30cm x 10cm in size.

²⁷ See Ollerton [1996].

Trust

It (the freedom to learn) aims towards a climate of trust in the classroom in which curiosity and the natural desire to learn can be nourished and enhanced. Rogers [1983:3].

Trust is built upon mutual respect. Earlier in this chapter I described the importance I place upon trusting pupils, of showing them respect, as a key ingredient of classroom atmosphere. I work explicitly on issues of trust and openly share my philosophy of learning mathematics and discuss the methods I use. I try not to hide my fallibility, the difficulties have, nor the mistakes I make.

Pupils trusting their teacher

The following anecdote from October 1993 is of a lesson where I had planned to introduce a Y10 class to graphic calculators. I knew exactly how I intended to begin the lesson and had arrived at school early in order to prepare work for the pupils. However, a number of events militated against my being able to carry out my plan, so I decided an appropriate and truthful course of action was to relate the following sequence of events to the class.

By 6.30am I had written feedback comments on three pieces of coursework. Next I began to go through my plans for the day; these included introducing *you* to graphical calculators.

I chose to explain what I intended them to achieve using a graphical calculator, these were:

- to key in instructions to generate sequences;
- to program a calculator to write and use formulae;
- to draw graphs and use the 'trace' function.

I also decided to give the class an insight into what my job involved.

I had planned to provide a brief introduction and use an information sheet²⁸ to help everyone make a start. I arrived at

²⁸ I distinguish information sheets, which help pupils to access information and develop broader problems, from mechanistic and repetitive worksheets, where separate skills are practised with great frequency, and in isolation to other skills.

school before 8.30am, to give myself plenty of time to make photocopies, however, neither photocopier was working. I therefore decided to write the information on the board prior to the lesson. However, this also wasn't possible because, due to a staff absence²⁹, I had to take a form group register

Although I didn't know the majority of the registration class very well, I tried to make the event a positive one. I did this by asking two pupils to assist me with the other pupils' names and by offering a cheery, enthusiastic "*good morning*" to each pupil upon calling their names. This despite the fact that several pupils looked anything but cheery or enthusiastic.

The bell goes and the group file slowly out of the room. Everything seems orderly, when suddenly a fracas breaks out between two boys. I decide to intervene and to calm the situation, I take them into the relative quietness of the Maths storeroom. By the time we reach a resolution, the first lesson has begun. I send the two boys on their way and open up the graphic calculator locker, to find four calculators missing. As my class are still arriving I quickly ask other maths teachers in nearby rooms if they know of the whereabouts of the missing calculators.

Having solved this problem I neither have an information sheet nor are there any instructions on the board. I therefore decide to use a graphic calculator designed for use on an over-head projector. I collect this from the store and return to the classroom; I place the OHP on a desk, pull down the screen, plug in the OHP, switch on ... and of course the bulb has gone.

Despite this false start to the lesson, the class were being quiet and patient. However, I was concerned to begin the lesson as quickly as I could.

²⁹ In 1993, because of the joint nature of my work with ITT institutions, I did not have a tutor group. I consequently filled in for absent colleagues.

I know there is a portable OHP just up the corridor in the library. I dash to get it, wheel it back to the room, place it in front of the screen only to find the cable isn't long enough to reach the socket.

The class continued to be patient. Having been thwarted at every turn, I still had not offered them a starting point; recognising I needed time to think of an alternative. With a deal of gesticulation, therefore, I described the above sequence of events.

This served the following purposes:

- it provided the pupils with an honest account of the difficulties I had encountered;
- by disclosing my fallibility I showed pupils trust; and
- I gave myself more time to decide how I was going to start the lesson.

The class listened attentively, and during this time my starting point emerged, this was to write the following on the board:

1 EXE Ans +1 EXE EXE EXE ...

This proved useful. Pupils worked on this and developed it with further suggestions. A little later Peter, a pupil with a record of some behavioural difficulties, borrowed an extension lead and I was able also to use the OHP. The lesson continued in a pleasant, productive manner and by the end some pupils were working on iterative sequences such as:

Input EXE Ans ÷3 +1 EXE EXE EXE ...

Through such incidents I intend to build up a sense of trust and honesty. I also want pupils to trust the tasks I offer. Gaining trust is also important to:

- avoid a false justification for why I wish them to learn certain skills;
- encourage pupils to explore ideas in depth;
- create a belief based upon "*I can do mathematics*"

Teacher trusting the pupils

Looking for opportunities to trust pupils is important. To illustrate this aspect of trust I use an anecdote from the final lesson of the term with a Y11 group in December 1991.

Jane is the strongest mathematician in my Y11 group. She pursues problems in-depth, and only occasionally do I need to ask her how she is "*getting on*".

The relationship I had built with Jane was based upon her autonomy to develop mathematics without my needing to overstate how to do something. Once I had posed a problem, she was capable of developing it in depth. I trusted her judgement to decide when to ask for guidance.

It is the final lesson of the Autumn term and Jane asks if I have an interesting problem for her to solve. This takes me a little off-guard as I was expecting her to continue her work on the current module. However, she has completed this. I decide to offer her a starting point for the module I had planned for the following term. I also invite two other pupils, Peter and Jackie, to work with Jane, and introduce them to the Glentop Graphical Calculus program³⁰.

Building up a mental profile of pupils and deciding who to offer extension tasks to typifies the kinds of decisions I make, particularly when working with pupils' differentiated responses in mixed-ability groups.

I ask two other people to work with Jane because I think they are capable of engaging with the complexities of gradient functions and because I feel they will make faster progress working together as a group than individually. I begin by reminding them about some previous work they had done in Y10 involving gradients of straight lines.

³⁰ The Graphical Calculus program was produced by David Tall; I first used it when studying the Open University ME234 course.

All pupils had explored $y = mx + c$ and some developed their work into quadratics. I decide to build upon this and extend their knowledge of drawing tangents to curves, leading to concepts of gradient functions. My exposition was quite didactic and, whilst I knew my explanations would not be fully comprehended, I needed the pupils to trust me enough to know that, as they worked on the task, connections would become apparent.

I show them how to access the Glentop program and how to determine the gradient function for $y = x^2$. Having established the result $y = 2x$, I leave them to explore other functions. As the computer is in the maths storeroom I decide not to wheel it through to the classroom; this means the three of them can work unsupervised for the remainder of the lesson.

Trusting pupils to work without constant supervision is necessary if they are to have opportunities to take responsibility and gain independence. When pupils exhibit these qualities and work in an atmosphere of trust, I can cede control. Working with pupils in a mutually trusting atmosphere, enables me to be at my most effective as a teacher.

Caring

And it occurred to me there is no manual that deals with the real business of motorcycle maintenance, the most important aspect of all. Caring about what you are doing is considered either unimportant or taken for granted. Pirsig [1974:27].

I care about what I do because I wish to be a good teacher. Caring about pupils and wanting to help them develop as rational thinking people is of paramount importance. Caring about how I organise conditions to create a supportive classroom atmosphere is central to effective learning.

The following anecdote from September 1991 describes a positive and caring atmosphere which another teacher constructed whilst simultaneously working on her children's mathematics. The anecdote describes events during the first three

weeks of a PGCE Secondary course at Keele University³¹. My role was to visit the students in their schools.

I knock on the classroom door and am invited in by the teacher. She is holding up a floppy rag doll and a Teddy Bear and asks the child by her side a number of questions: How old was she? When was she first given the toys? Did she still love them? Did she still cuddle them in bed? How old were the toys? The rest of the class of six and seven year olds are listening attentively; they are invited to respond to the teacher's question of how old their own toys are. Answers are confirmed by the teacher drawing a number line on the board and counting on from one age to another.

The atmosphere the teacher created and the way in which she skilfully encouraged the children to attend to the mathematics, of the age of teddy bears, was based upon care. I also found myself being fleetingly aware, in those moments, about how interested I had become in the ages of Teddy Bears. Her caring approach was clearly transmitted to the enthralled children; she created a caring climate within which the pupils engaged with numerical skills. The social context, based upon Teddy Bears, and the fact the children had 'ownership' of their bears was clearly significant in creating an effective learning environment. Creating environments where love and care are synonymous with learning is a key issue. Papert similarly describes Turtle geometry as a context in which children can engage and enjoy the mathematics they are doing.

a piece of learnable and loveable mathematics. Papert [1972:238].

Issues of love and care are not, of course, to be confined to the lower primary age range. It is equally significant in the secondary classroom. In her tribute to Simon Haines, who died in March 1994, Watson [1994:22] described her first meeting with him as follows:

³¹ Students were placed in a Primary school in order to gain a perspective on child development and to observe children in classroom contexts which would differ from those they would meet later.

I had the privilege of interviewing him for his first job. I asked him: "what would I see if I came into your classroom?" and he replied "Love". This might sound a bit sloppy but it turned out to be true. It was a tough kind of love and it worked.

To illustrate further the importance of constructing a caring atmosphere, I offer the following descriptions of two students' lessons, each of whom brought qualities to their lessons which caused the outcome of their lessons to be very different. The first lesson, from May 1994 describes a student, Liz, using "Numerator", an interactive maths program, with a Y8 class.

Liz has set up the task and is speaking in a pleasant 'matter of fact' manner. Her approach automatically assumes the pupils will engage on the task without fuss. As she moves around the group it appears she really wants to engage with them. She shows care for the pupils and the atmosphere in the room is friendly and work-like.

Liz exuded a caring approach; her pupils responded well and engaged purposefully with the work. It was upon such evidence that her mentor and I agreed she had a deal of potential.

The second lesson with a Y9 class was taught by Tim, another student. The content of his lesson was 'Adding Fractions'.

Tim writes his method for adding fractions on the board. He actively ignores statements from some pupils who say they already know how to add fractions.

The style was entirely teacher-centred. Tim clearly had a script from which he did not intend to shift. Unless a response fitted his predetermined method, any other contributions were unwelcome.

Some pupils know about the fraction key on their calculators and because they are using them to answer the questions from the text book, Tim takes the calculators from them.

There were no positive interactions between student and pupils, and the atmosphere was of teacher and learners barely tolerating one another. For these pupils, in this lesson, learning mathematics was about following a set procedure in order to carry out an exercise which offered little challenge.

In these two classrooms there were different degrees of care and mental liveliness. The first student showed a "tough kind of love", she expected her pupils to respond and through her interactions, she demonstrated care. In the second classroom there was little sense of the teacher caring either for the pupils or for the mathematics; consequently the pupils showed little care in return.

Caring about pupils, about how to make mathematics accessible, and to capture pupils' imaginations is an important link between teaching and learning. Rogers is clear about the impact a caring teacher can have upon children learning basic skills. He draws upon "massive" amounts of research data as follows:

In farflung studies involving hundreds of teachers and thousands of students from primary grades, through technical schools, massive data has been accumulated. Very briefly, their (Aspey and Roebuck, Reinhard and Anne-Marie Tausch) work shows clearly that when a teacher is real, understanding and caring, students learn more of the "basics", and in addition exhibit more creativity and problem-solving qualities. Rogers [1983:3].

Teachers cannot legislate for the range of responses, or explain why some pupils behave in certain ways. It is not possible to know the many pressures and problems some pupils are under, and teachers are not miracle workers. I am often 'brought to ground' by certain responses from pupils, two examples are:

Peter is a year 8 boy. He frequently seeks my attention often, I feel, in negative ways; during one lesson I notice he has been crying, at first I chose to not to respond to him. However as the lesson progresses Peter becomes more distressed. I eventually I ask him to explain his upset to me; it transpires he is worried about his father who is in hospital for an operation.

Stuart, is a Y11 pupil, He has arrived in my class having been expelled from several other schools for behavioural reasons; he is currently awaiting a court appearance on an ABH charge. I make a point of showing him care, and this is reciprocated in his friendly behaviour and by his unusually good attendance.

My ability to form a sound relationship with Stuart had positive effects on some of the more disillusioned pupils in the group; this was because of the reputation he had arrived with, which appeared to be contrary to the friendly and amenable behaviour he exhibited in my classroom.

...the facilitation of significant learning rests upon certain attitudinal qualities that exist in the personal 'relationship' between the facilitator and the learner. Rogers, C. [1983:121].

A phrase Stuart once used as I feigned annoyance and, which I shall long remember, was: "*Chill out Teach*"

And I did.

In completion of this section, I illustrate the need pupils have for their teachers to care for them. I describe a particularly moving event which occurred on several occasions between myself and Sarah, a Y11 pupil. Sarah struggles to understand complex mathematical concepts; were the department to practice setting, she would most certainly be placed in the 'lowest' set. I base this statement upon my knowledge of having taught her for the three previous years and recognising how she frequently struggled to engage with mathematics in depth.

In the dynamics of the un-setted group, Sarah 'holds her own'. She is quite explicit about asking for help; I often hear her explaining to other, supposedly 'more able', pupils what they should be doing, particularly if she feels they are not applying themselves to a task as well as she thinks they ought to.

The way Sarah worked and responded to difficulties provided other, more fortunate pupils, with a salient model for working together in a non-segregated classroom. Because the emphasis was upon pupils learning mathematics in a caring and socially integrated context, it did not matter that pupils were achieving

different levels of understanding. Indeed the expectation that this would happen as a natural course of events became an incentive for pupils to recognise their responsibility for pursuing their mathematics as far as they were capable³².

Sarah receives abuse counselling and as a consequence school is very important to her; she is cared for and looked after. Sometimes she has to leave my lesson early for an appointment with her counsellor; she knows I am aware of the reason for her to leave my lesson early. At the beginning of some lessons, Sarah links arms with me, as a way of saying "hello" and wanting a moment's attention. This show of affection is carried out quite openly as other members of the class enter the room.

Because of the public nature of this event I did not feel compromised or that my professionalism was being brought into question.

I usually ask her if she is alright and if she knows what she is doing with regard to her work. She usually replies: *"Yes I know what I'm doing but if I get stuck I'll ask for some help."*

Events such as this provide a poignant reminder of the kinds of pressures some pupils are under and the enormous responsibilities teachers have for helping them develop, both academically and socially. They re-affirm why I am a teacher and reinforce the importance of creating a classroom atmosphere, where learners feel valued, cared for; that what they do and how they do it really does matter.

Occurrences of this nature confirms that teaching transcends the conveyance of knowledge and skills and highlights the value of celebrating individuals' successes, their contributions, and their differences. As such, teaching is a privilege.

³² I develop this issue of less confident pupils operating alongside their more confident peers in the 'Un-setted' chapter pp. 183-185.

Summary

In this chapter I have demonstrated how a positive classroom atmosphere supports learning. I have described aspects of my teaching styles and the classroom culture I seek to create. The anecdotes I have drawn upon are only fragments from a few lessons and as such are but a minute part of my total experience as a teacher; the issues I have drawn out represent the importance I place upon classroom atmosphere.

Integral to this chapter is my recognition and definition of the elements implicit in my classroom atmosphere. Through my analysis of anecdotes I have demonstrated how these elements have a powerful effect upon pupils' learning. I have described the importance of displaying pupils' work, of constructing a happy and challenging environment and the positive effect this has upon pupils' confidence. I have shown how learning can be enhanced through encouraging greater pupil responsibility and ownership of the work they do, and how this can be achieved through co-operation and trust.

Although I have undergone processes of recognising and describing key elements in my own practice, and have identified how these elements support children's learning, I do not suggest that other teachers should implant similar elements into their classrooms. Others may choose to analyse their own practices, however, because teaching is an idiosyncratic and individualistic craft, transferring one teacher's style to another's practice requires careful consideration. What is transferable is the process of reflecting upon practice, to consider the techniques, skills, rationales, aims and objectives which are important and common to all teaching. These can be debated and developed if we are to be effective in enabling children to move from childhood into adulthood as discerning, confident and caring citizens in an even more demanding world.

I conclude that to analyse what we do, the way we choose to respond to pupils, and the atmosphere we seek to create has a profound effect upon pupils' learning opportunities and the learning outcomes achieved.

Chapter 4

Curriculum Development

Reflecting upon, evaluating and sharing practice

Curriculum change does not occur independently of a change in teachers' beliefs about the nature of mathematics, the nature of mathematics learning and the nature of mathematics teaching. This reconceptualization by teachers is an integral part of curriculum innovation and serves as a driving force for change. Furthermore, teachers are the essence of the innovation process and should play a major role in material development and curriculum planning, as well as being involved in the evaluation of the effectiveness of new materials and methods.

D'Ambrosio, B.S. [1991:84].

Outline

This chapter is written in two sections. I describe how and why I engaged with curriculum development and the process of implementing change. The first section is a historical perspective detailing my changing practice. I describe key events and influences which shaped my philosophy and practice. These were:

- from 1973 - 1985: working in an innovative mathematics department, helping to develop a lower-school modularised curriculum framework and upper-school mode-3 CSE syllabus;
- from 1986: becoming a head of mathematics department; re-working existing schemes of work; moving away from published schemes towards a problem solving methodology; developing a central resource area; altering the use of additional support time for pupils with learning difficulties; joining the ATM-SEG GCSE syllabus group¹.

¹ The Association of Teachers of Mathematics and the Southern Examining Group. See appendix 3a for a description of how ATM-SEG GCSE group formed and developed.

In the second section I explain my rationale for working on curriculum development, and the significant goals for change I worked towards. These changes involved a shift of methodology regarding pupils learning, from atomised^s content skills^s, based upon text books and repetitive worksheets, to a problem-solving approach where content skills were developed within the context of the problem being solved. I describe how this led to the creation of a modular curriculum built upon investigative approaches^s to aid learning. I explain how these learning styles were legitimised, particularly between 1986 and 1990, through broad-based GCSE assessment criteria developed by the ATM-SEG group and how this further supported curriculum development by collaboration with six other schools who formed the ATM-SEG GCSE cohort.

I describe how these developments, based upon accessible starting points for each module, supported differentiated learning outcomes, and became crucial to the fundamental principle: *...extensions rather than deletions*. DES [1985a:26]. Finally I address issues of risk-taking. I discuss the importance of this, both for my own practice as an established teacher, and for initial teacher trainees.

History

By reflecting in the present I recognise some unchallenging lessons I have taught in the past. Consequently I perceive substantial changes in my practice. These changes have arisen through many influences which, in broad terms, were shifts from teaching fragmented, single skills to constructing a holistic, modularised curriculum structure. The following quote from HMI was one such influence:

Much of the mathematical experience of most pupils is extremely fragmented, as they proceed from one small item to another in quick succession. Indeed because of the commonly held view that 'many pupils cannot concentrate for any length of time,' many textbooks are planned to provide this rapidly changing experience... An in-depth study is of potential value for all pupils, not only mathematically but also in terms of the development of personal qualities such as commitment and persistence. DES [1985a:6].

Wyndham Mathematics Department (1973-1985)**Lower school schemes of work: developing a modularised framework with mixed-ability groups**

I first encountered modular curriculum design at Wyndham School, Egremont, Cumbria, in 1973 under the visionary and inspirational leadership of Eric Love, then Head of Department. He created a department ethos grounded in curriculum development and based upon investigative approaches. This was recognised by the Schools Council [1977:79], where the following description of the department in a report on mixed-ability teaching in mathematics appeared:

A high level of professional concern and commitment characterized the department. Mixed-ability methods in the first and second years were being developed in an atmosphere in which teachers were encouraged to work together, to talk and to share experiences... The head of department had established a philosophy of mixed-ability teaching before developing the organization to put it into operation.

Love sought to develop modules for lower school classes to replace the original SMP work card scheme, previously adopted as a stop-gap measure; each module was based upon a variety of resources rather than a specific text. The resources we used were taken from publications by:

- The Association of Teachers of Mathematics;
- Dime materials;
- Leapfrog booklets; and
- The South Nottinghamshire project.

The latter project was created to support whole class mixed-ability teaching and as an alternative to pupils following individualized schemes:

This project accepts the principle of the whole class working in the same general field but with differentiation of the particular problems attempted by individuals. It rejects individualized, independent learning schemes as a strategy for teaching mixed-ability classes. Bell et al., [1975:140].

Other texts which department members were encouraged to use were: 'Starting Points'² and 'Making Mathematics'³. As ideas and materials were tried out, they were written into a termly, then a yearly planner based upon fortnightly units of time. The work of the department, particularly the investigative approaches, was recognised and validated by the Cockcroft commission when members of the committee visited the school in 1979⁴. A decade later such ways of working emerged in National Curriculum as the Ma1 Attainment Target - Using and Applying mathematics.

Upper school mode-3 GCE and CSE syllabuses

The department developed a mode-3 GCE O-level, and two mode-3 CSE syllabuses. At O-level, pupils investigated numeric, geometric and strategic⁵ problems. CSE coursework, focused on: numeric/algebraic, geometric, statistical and applied⁶ problems. Each problem lasted for two to three weeks. Over the 2-year course pupils did five such pieces of work accounting for 33% at O-level and 40% at CSE of the overall grade. I took increasing responsibility for developing the CSE syllabuses and whilst investigations became the assessment vehicle for coursework, investigative approaches began to influence the more traditional content-focused aspects of the curriculum. The structure and rationale of the Wyndham framework under-pinned curriculum development and design at Orleton Park.

² Banwell, C., Saunders, K., and Tahta, D. [1972] Starting Points, Oxford University Press.

³ Paling, D., Banwell, C., and Saunders, K. [1969] Making Mathematics, Books 1-4, Oxford University Press.

⁴ Wyndham mathematics department was one of the schools whose approach to investigative work had a direct influence on Cockcroft 243.

⁵ A typical strategic type problem was to explore ways of visiting every square on different sized 'chessboards' using knights moves.

⁶ These problems focused on skills such as planning, sequencing and costing, for example setting up a small business or organising a trip. They were early attempts to engage pupils in what is now known as 'Mathematical Modelling'.

Becoming a head of department at Orleton Park School (1986)

I received massive support and encouragement to change practices in the mathematics department, from the senior management team, two of whom taught mathematics. As head of department, I perceived my responsibilities towards pupils and colleagues ran in parallel; the common denominator being curriculum development. I adapted the Wyndham model of a planned matrix of topics and looked for more problem-solving tasks to offer accessible starting points for each module. This was a fundamental cornerstone.

I began to construct files of teachers notes to describe starting points, supplementary ideas and extension tasks for each module, and for each year group. A synopsis for each module was produced for speedy reference⁷. New ideas were written up and collected together and distributed using plastic A4 wallets⁸. Sharing anecdotes from lessons and recording ideas and became a vital part of the process of development.

The planner and synopsis were up-dated each year; this process was carried out each Summer term when members of the department were re-circulated with the current synopsis and asked to add new ideas they had found successful. Ideas were then incorporated for the following year. Curriculum development was, therefore, a shared, on-going process, which encouraged teacher involvement.

Whilst it was important to share ideas and resources, to avoid duplication, and to inform department staff of the general progression of modules, strict adherence to the planner was not intended; the planner existed as a supportive framework, to enable flexibility rather than to stifle innovation.

Recording ideas to aid curriculum planning

A typical example of curriculum development, from 1993, centres on a colleague, planning to leave work for a class whilst he attended a meeting. He had begun a

⁷ Extracts from the 1992-93 department scheme of work, the Y7 and Y10 planners, together with the synopsis of ideas for each module appear in the Open University PGCE [1993: pp. 15-22].

⁸ These wallets are a marvellous innovation for organising, distributing and storing ideas.

module based upon Vectors in the previous lesson and wished to provide his pupils with a problem they could work on and extend without needing specialised help from the supply teacher. He knew the class had previously met the Snook's problem so he suggested the idea of exploring the paths made in terms of vectors, thereby developing pupils' understanding of vectors within a recognisable context. This idea was subsequently written into the following year's synopsis and so the curriculum was developed.

There were three further significant events to aid change:

- creating a central resource area for the mathematics department;
- altering the use of 'remedial' support time; and
- joining the ATM-SEG GCSE syllabus group.

Creating a central mathematics' resource area

By partitioning off a large, central corridor area⁹ we were able to store and access resources easily and more equitably. Previously teachers had stored resources in their own classroom cupboards, and this led to an unsystematic, first come, first served basis; which created obvious difficulties for sharing resources. Once the store had been built I purchased a range of basic equipment, such as multi-link cubes, Dime materials, Cuisenaire rods, ATM MATs and class sets of simple, scientific and, in the fullness of time, graphical calculators. I had previously made two class sets of 9-pin and 25-pin geoboards, and pegboards using scrap wood from a timber yard.

The implications of equipping the department with such resources, was to divert capitation away from buying text books and other published materials. The process of encouraging colleagues to be less text book dependent and instead adopt problem-solving type approaches, was lengthy and occurred concurrently with decisions to move from setted to un-setted teaching groups.

⁹ The building work was carried out by Dave Ridgway, deputy in the mathematics department.

Altering the use of additional pupil support time

During my first two years as HoD I re-structured the timetable allocation of support time, and changed its use from supporting children with specific learning difficulties to supporting teachers in their classrooms. This proved a significant vehicle for change. My reasons were as follows:

- I did not perceive such time was used to the greatest effect. Support lessons were often fitted in to 'make up' teachers' timetables and subsequently became a very low priority;
- I did not feel the children, for whom the support was intended, gained benefit from having another teacher sat next to them in the classroom. Indeed, at worst, this proved counter-productive as it undermined the confidence of the very children for whom the support was intended;
- I could use this time to work with other teachers in their classrooms to develop different, additional teaching styles to the predominant didactic, exercise-driven methodology;
- By developing a wider range of teaching styles all children would benefit in the long term.

During my first two years, therefore, up to one third of my timetable was used to work with other teachers. This enabled me to either do lead lessons² or to team-teach. An essential component of this process were further discussions about possible developments which could arise from a starting point.

Joining the ATM-SEG GCSE syllabus group.

Although I had previously been involved in curriculum development through InSET, I could not have foreseen the depth and quality of personal development I was about to engage with in March 1986, as a result of becoming a pilot school for the Association of Teachers of Mathematics - Southern Examining Group (ATM-SEG) GCSE syllabus.

The first drafts of this syllabus were written by members of the ATM, and my decision to join this group were based partly on the following sequence of events:

Having arrived at the school in January, one of my early tasks was to mark the mock exams of the 'bottom set' Y11 CSE group¹⁰. The deep dismay I felt as I looked at pupils' pitiable attempts to answer questions, the majority of which made little or no sense to them, was to have a profound effect upon my immediate actions as a head of department and my future writing¹¹.

This event became a strong motivator for me to seek out a GCSE syllabus which offered pupils a form of assessment which transcended a specific, artificial testing methodology.

I find it an appalling notion that anyone believes they can write questions that are going to assess, effectively and fairly, large numbers of children throughout the country... doing tests... based on certain spurious levels... on the same day... at the same time. Ollerton [1993:27].

The situation of so-called lower attaining pupils, being failed by the system of examinations, was not, sadly, going to change.

The 1993 OFSTED GCSE report notes that many pupils gaining lower grade results make little attempt to answer any of the questions on the paper - they appear to give up at the first hurdle... The design of questions is cited by some respondents as the reason why weaker pupils do not demonstrate what they can in fact understand. Ofsted [1994:24].

The ATM-SEG 100% coursework based GCSE syllabus (1986 to 1989):

A community of schools

Sharing the construction of the ATM-SEG GCSE mode-3 syllabus was the most intense phase of curriculum development I engaged with. Pupils work was

¹⁰ From the outset I gained agreement with Paul Smethurst, who taught the next-to-bottom set, that we join our two classes together and team-teach them. This provided continuous opportunities for on-going support and sharing ideas.

¹¹ See Ollerton [1991] and [1993].

assessed against nine domains, later reduced to seven; Mathematical knowledge¹², and six further domains. These were: Communication, Implementation, Interpretation, Mathematical attitude, Autonomy and Evaluation¹³.

The opportunities this radical syllabus offered, as a catalyst for change and innovation, was phenomenal. The drive, the sense of purpose and the strength of community was such that teachers from each school, moderators, original syllabus writers and officers from the SEG attended weekend conferences every term during the period when the syllabus existed in its 100% coursework assessed format. These gatherings¹⁴, funded by the ATM and SEG, enabled up to twenty people to work together.

The energy these remarkable meetings generated was highly motivating. Whilst the purposes of these weekends were intended for inter-school cross-moderation of teacher assessment, and to monitor syllabus development, the underlying focus was professional and curriculum development. By moderating pupils' work from other schools, we inevitably recognised new ideas which led to discussions about how one teacher had set up a particular sequence of lessons.

The ATM-SEG GCSE was an extraordinary, collaborative experience. There developed a common bond of experience and responsibility for:

- creating methods of assessment which transcended the limitations of terminal examination;
- supporting a variety of ways of teaching mathematical concepts;
- acknowledging and celebrating pupils' achievements in various ways.

Each of these points proved to be prophetic:

Our aim... must be to develop an approach to the curriculum in KS4 which seeks to develop the talents of all students; which recognises the multi-faceted nature of talent; and which accepts that, as a nation we have for too long had too limited a concept of what constitutes worthwhile achievement. Dearing [1993:43].

¹² This was defined by the content list in The National Criteria pp. 3-4: DES [1985b].

¹³ For the remaining six ATM-SEG GCSE grade descriptors see appendix 3b.

¹⁴ Most of these meetings took place at hotels and the ATM offices in Derby.

Working with this group of teachers, using coursework as the central plank, was a highly motivating experience, which reflected in our teaching and pupils' learning:

there is evidence of the beneficial effect that coursework has had on the motivation of pupils, especially the more able and those contemplating studying GCE A Level. Ofsted [1994:24].

Changes to the ATM-SEG GCSE syllabus (1990 to 1992)

From 1990, at SEAC's¹⁵ behest, the ATM-SEG syllabus assessment profile was changed to 50% coursework, and 50% examination. The effect of imposing an examination, upon the group, was to divert energies away from curriculum development, to writing examination papers¹⁶. Furthermore, the high standards that existed within schools, that pupils' folders must demonstrate curriculum coverage, was severely reduced, and became comparatively lightweight.

The demise of the ATM-SEG syllabus (1992 to 1994)

In 1994 the ATM GCSE group disbanded. This was as a direct result of every GCSE mathematics syllabus being reduced to a maximum of 20% for coursework¹⁷. The marginalisation of teacher assessment, at a national level, left the group without a realistic brief for continuation. All teachers, no matter what skills they had developed, were bound by the same, narrow assessment criteria. Teachers in the ATM schools, having worked with a high level of motivation, in terms of curriculum development and assessment methodology, had lost the impetus to continue.

This downgrading of teachers' professionalism parallels issues of falling standards in classrooms as a result of teaching to the middle. Freudenthal is unequivocal in his criticism of national tests, referring to them as *the big lie* within education:

¹⁵ Schools Examination and Assessment Council.

¹⁶ The group spent many hours constructing examination papers in order to meet the requirements laid down by SEAC. Indeed a significant proportion of time was taken up with meetings to write papers at the 1989 ATM Easter conference.

¹⁷ This followed John Major's speech at the Cafe Royal to the Centre for Policy Studies in June 1991.

Tests have made it clear beyond any doubt that the vast majority of pupils have not the slightest idea of what is meant by this highly sophisticated mathematics they are supposed to have learned... I like to call it the big lie of our educational system... The wide gap between what is claimed to be taught and accepted as being learned -- the big lie -- is inherent in any system that strictly relies upon exterior control. The gap is reduced or absent at institutions that are trusted to set standards of their own. Freudenthal [1991:166].

Rogers [1983:21] similarly describes the undermining effect, upon teachers and learners, of prescribing the content of what is to be taught and how it is to be assessed:

When we put together in one scheme such elements as a 'prescribed curriculum', 'similar assignments for all students', 'lecturing' as almost the only mode of instruction, 'standard tests' by which all students are externally evaluated, and 'instructor-chosen grades' as the measure of learning then we can almost guarantee that meaningful learning will be at an absolute minimum.

Indeed Ofsted also recognised the problems of such centralisation:

Some teachers and other commentators feel that the National Curriculum has diverted the attention of the best innovative teachers and sapped their enthusiasm. Ofsted [1994:35].

Curriculum development occurred, therefore, within the department and as a result of working on a shared task, with colleagues from other schools, through the ATM. The chief motivation for curriculum development arose as a result of teachers in the ATM-SEG schools having greater autonomy over, and increased responsibility for, assessment procedures.

Rationale

My rationale for curriculum development is based upon improving ways pupils learn, and personal renewal. Maintaining an interest in developing the ways I teach to increase my effectiveness and to enhance the learning experiences of pupils, students, and colleagues is paramount.

For pupils I look for opportunities to nurture differentiated learning outcomes, without needing to segregate them into notional 'ability' groups.

For colleagues, as an HoD, I sought to create a shared, co-planned and coherent framework which staff could take responsibility for and ownership of.

As an initial teacher trainer I seek to help students develop their styles and equip them with a range of teaching strategies which they can apply to classes, settled or not; with other tutors I wish to share ideas which, in turn aids students to be more explicit about their learning experiences¹⁸.

The following describes the framework of my practice and are my sub-headings for the remainder of this chapter:

- constructing a modularised framework;
- developing story lines: starter and next tasks ;
- using a range of resource materials;
- using investigative, problem solving approaches;
- finding alternative approaches to using published schemes or repetitive worksheets;
- taking 'risks' to open up new possibilities.

Constructing a modularised framework

Module titles were as broad as Transformations or Trigonometry. Having decided upon the module and the year group, my planning was based upon the following questions:

- What specific skills do pupils need in order to access broader concepts?

¹⁸ An example of this is the use of journals for PGCE students at UCSM, Lancaster.

- What starting point can I use to enable this access and cater for pupils' different learning paces and different depths of learning?
- What strategies can I draw upon when setting up the starting point?
- What resources might I use?
- What extension ideas or next^s tasks can I offer pupils once they have demonstrated understanding as a result of working on a starter task?
- How might I encourage pupils' to use and develop process skills such as: organising, specialising, generalising and proving, within this framework?

In my detailed planning I shift from a broad overview to a narrow focus. I peel away my layers of knowledge about a particular concept and re-construct what I know about the kernel. This forces me to focus on specific learning objectives. In effect I return to my basics. Deconstructing one's knowledge and deciding where to begin, in order to provide pupil with access to concepts, can be applied to all of the curriculum; this planning process is supported by Kaput [1994:77]:

In order to do this (teach calculus) I am forced to examine the nature of the mathematical content of calculus: What is it? What are its objectives, methods and especially its representations? And in order to do this, I am further required to look back...

A consequence of this process is that it has caused me to engage with questions of justification relating to:

- Why learn about transformations or trigonometry?
- Why learn mathematics?
- Why teach mathematics?
- Why teach?

Considering such questions is central to my thesis and to the development of my rationale of how and why I teach the way I teach.

The above framework is under-pinned by the following principles:

- The effectiveness of a starting point for a module is gauged by the way it offers all pupils access to concepts, by requiring and building upon small amounts of pre-conceptual knowledge.

- All pupils will have opportunities to move speedily from the starting point, and develop their thinking in ways commensurate with their potential.
- Pupils will work on the concepts over several lessons, thus enabling pupils to develop understandings to different depths.
- Interdependent skills, subordinate^s to the main concept, are met, practised and consolidated within different modules, thus pupils will see common skills emerge in various contexts.

Each module title is a place-holder where content skills are practised, and concepts are developed, and connected together. 'Transformations', for example involves pupils working with: co-ordinates, negative numbers, reflections, rotations, translations, enlargements, area, equations of lines and matrices.

Different modules provide learners with opportunities to engage with common and overlapping skills which they have met elsewhere in the curriculum. This is consistent with my view of the inter-connected nature of mathematics.

Further modules are described elsewhere, i.e. 'Volume': Ollerton [1994:70], and 'Trigonometry': Ashworth [1995:9].

Developing story lines: starter and next tasks

A story line describes an initial starter task together with a list of possible next tasks. This principle of providing all pupils with opportunities to develop concepts to varying depths, according to their interest and potential, is a process of "seeding" Ollerton [1991:32].

Developing story lines aligns itself closely to scaffolding, a process defined by Wood, et. al., [1976: pp 89-100] in terms of a child or a novice who can:

... solve a problem, carry out a task or achieve a goal which would be beyond his unassisted efforts... the process can potentially achieve much more for the learner than an assisted completion of the task. It may result, eventually, in development of task competence by the learner at a pace that would far outstrip his unassisted efforts.

The starting point and further tasks for explorations become the scaffold. However, not all the pupils will extend their thinking to the same depth along my planned story line; they will have different existing knowledge and will have formed different constructions of knowledge according to prior experiences. It is necessary, therefore, to provide everyone with opportunities to extend their thinking as far as possible. This is a further example of differentiation; the connections, understandings and internalisations that occur for different children are difficult to predict and impossible to quantify; thus the frailty of traditional paper and pencil tests as way of measuring children's ability. What is certain is different outcomes will occur; by planning for differentiated outcomes, therefore, I address issues of equality of opportunity.

To illustrate this I describe my planning behind the 'Transformation' module. My ultimate aim is for pupils to learn how to transform shapes by matrices; the narrow focus is an exploration of the effect of reflecting and rotating simple shapes drawn on a co-ordinate grid. for which I provide pupils with pegs and pegboards, grid paper and tracing paper. Following a whole-class introduction I set the task of exploring how the co-ordinates, which define the corners of a shape change after a reflection in the x -axis

I can predict some pupils, although not necessarily who, will quickly complete this task and will be able to write an explanation to describe how the general point (x, y) transforms to $(x, -y)$. Extension tasks are to consider other reflections through $x = 0$, $y = x$ and $y = -x$. Further tasks are to explore rotations of 90° , 180° , 270° and 360° about the origin.

As pupils complete these tasks I have more problems for them to solve, e.g. to consider combinations of transformations and eventually explore transformations using 2 by 2 matrices.

In the following section I discuss the types of practical equipment I use and my rationale for using such manipulatives.

Using a range of resource materials

For each of the modules, Transformations, Trigonometry and Volume, I use different types of practical equipment: pegs and pegboards, cardboard and scissors, and multi-link cubes. I use such equipment to:

- provide pupils with concrete experiences, something they can physically manipulate in order to construct their understanding of abstract concepts;
- offer pupils a focus of interest to provide them with a richness beyond my verbal or written communication;
- create an image upon which learners can, at a later time, return to in order to reconstruct their knowledge;
- show pupils that mathematics is something they can create and recognise that it exists beyond the pages of a textbook;
- provide pupils with imaginative approaches to learning mathematics and create within them an investment in mathematics.

This final point is endorsed by Watson [1994b:56], where she describes her observation of Russian pupils who had been asked to carry out a homework task, which was to make skeleton models using any raw-materials:

The time and emotion invested in the structures must have been enormous. They were proud of their creations and eager to work with them.

The use of practical equipment, and the range of materials which might be used, is supported and documented in the Open University PGCE course E884:

Often a concrete representation can be all that is needed to help a pupil make sense of a concept that had previously appeared difficult. However, it is important to help pupils connect the symbolic notation with the concrete representation for understanding to be improved. Selinger [1993:5].

It would, however, be complacent to deduce that different types of practical equipment are imbued with mystical qualities which automatically offer learners' insights into mathematical concepts; as Holt [1982:138] commented about Cuisenaire rods:

"The beauty of the rods..." I am very sceptical of this now. Bill (Hull) and I were excited about the rods because we could 'see' strong connections between the world of rods and the world of numbers. We therefore assumed that children looking at rods and doing things with them, could see how the world of numbers and numerical operations worked. The trouble with this theory was that Bill and I 'already' knew how the world of numbers worked. We could say "Oh, the rods behave just the way numbers do." But if we 'hadn't' known how numbers behave, would looking at the rods have enabled us to find out?

Practical equipment is only a means to an end. The learner must, at some point in their development, detach her/himself from the manipulatives in order to become independent of the scaffold, if s/he is to grasp the concepts which the teacher intended they understand at the outset.

... helping the human learner achieve mastery of his own without ending up dependent on the sources from which he has learned, I suspect that the answer is twofold. On the one hand, there is the cultivation of a sense of autonomy and competence in the broadest personal sense... But more than that, there is a question of whether what the individual has learned equips him to generate knowledge and opinions on his own. Bruner [1972:123].

Pupils must know what it is they have been learning, and be encouraged to be explicit about what they have learnt¹⁹. There is a significant difference between pupils 'doing' mathematics and understanding what it is they have done and learnt. Forcing such awareness is to place greater emphasis upon pupils' acknowledging their responsibility for their learning.

¹⁹ I develop explicitness in the 'Teaching and Learning' chapter pp. 133-137.

Using investigative, problem solving approaches

'Problem Solving' and 'Discovery Learning' have become catchwords. I never liked them as mere slogans, and I like them even less since I first saw them exemplified. Problem Solving: solving the teacher's or the textbook author's or the researcher's problems according to methods they had in mind, rather than the learner grasping something as a problem. Discovery learning: i.e. uncovering what was covered up by someone else -- hidden Easter eggs. Freudenthal [1991:46].

Whilst the most important aspect of my planning is to look for potentially interesting problems with built-in opportunities for differential outcomes, it is important to recognise that a starting point is not in itself sufficient to achieve differentiated outcomes; this must be supported by teacher interventions which, in turn, depend upon pupils' responses to the initial stimulus. Too often, I feel, open-ended work is misconstrued as a separate part of curriculum where pupils are expected to discover ideas for themselves, with hints from the teacher; this is due, in part, to the way such tasks are used for assessment purposes at GCSE.

I am equally aware of having used tasks, such as finding all the pentominoes², where pupils find all the possible shapes which can be made by joining five equal sized squares by their edges. Having 'discovered' there were exactly twelve possible solutions, I would ask pupils to solve puzzles which involved fitting the pentominoes together to make other shapes, or produce tessellation designs. Although this task provided pupils with opportunities to work systematically, it was only later in my development as a teacher I recognised such a task could provide the context for pupils to work on other concepts, such as:

- the symmetries of each shape;
- the perimeter of each shape;
- ideas of maximum and minimum;
- determining which shapes would fold up into an open box.

...simply developing a collection of interesting activities is not sufficient. The knowledge gained must lead somewhere. Thus what is constructed by an individual depends to some extent on what is brought to the

situation, where the current 'activity' fits in a sequence leading towards a goal, and how it relates to mathematical knowledge.

Romberg [1994:300].

As I developed my pedagogy I used investigative approaches in order to help pupils' develop their understanding of mathematics, rather than 'doing' an investigation as an end in itself. I encouraged pupils to use process skills to deepen their understanding of the content, see Ollerton [1993b: pp.26-29].

Schifter and Twomey Fosnot [1993:9] offer a similar perspective on how the tasks teachers provide must offer learners new insights:

When teachers are no longer sources of mathematical truth, but creators of problem-solving situations designed to elicit the discovery of new conceptual connections and new understandings, then it is especially important that they select tasks whose completion requires a cognitive reorganization. Thus will their student be called upon to use extant knowledge in new ways and to alter or expand their current stock of understandings.

A further example is exemplified in a module I titled 'Perimeter and Area', a collection of ideas about exploring shapes with constant perimeter and different areas. I have used this many times with GCSE and PGCE groups, although the initial introduction has varied. One starter task is to create a large space in the middle of the classroom and give each pupil a 30cm ruler, defined as a unit length (a plastic straw of a similar length would be equally useful). I ask the pupils put their rulers on the floor to form a rectangle then discuss the dimensions, the perimeter and the area of the rectangle²⁰. Other rectangles are made and this continues until several rectangles have been made. Pupils record results and look for any missing ones using integer values.

²⁰ I adapted this idea as a result of discussions with Anne Watson during an ATM Easter conference.

Extensions from this starting point are:

- finding dimensions to maximise area;
- using non-integer values for length (l) and width (w);
- graphing l (or w) against area;
- constructing formulae for perimeter and area using l and w ;
- changing the perimeter to another value then comparing results;
- choosing a constant area to explore rectangles with different perimeters;
- graphing l against w for the previous problem;
- constructing a formula to minimise perimeter;
- finding rectangles whose perimeter value equals the area value²¹.

This epitomises my planning for teaching, although the detail may vary, for instance, instead of deploying the active type of approach described above, I may explain the problem on the board, or use match sticks on an OHP. How I set up the initial task will depend upon a variety of factors such as: size of classroom, how well I know a class, the risks²² I feel I can take, and the kind of atmosphere I wish to create.

The following anecdote from a session with a PGCE group in April 1996 relates my latest development with such perimeter/area ideas. The session title was "Writing classroom materials", and I had decided to offer the students a range of problems and then ask the students to consider the kind of questions they might pose as extensions tasks.

I arrive at the allocated room to find another group are in situ and the tutor is sifting through some OHT's. This double room booking means I have to find another one. Having ascertained none are available we go into an empty lecture theatre. My plans for the session are changing by the moment. Originally I had intended to use the "Dance Squared" films, however given it has taken me a few minutes to find the master light switch, I decide against any further dealings with technological devices.

²¹ For example a 6 by 3 rectangle has an Area of 18 units and Perimeter of 18 units.

²² I develop issues of risk taking later in this chapter pp. 105-110.

One student suggests we utilise the floor space in front of the banked rows of seats, so we sit in a ring. This is an opportune moment to describe the constant perimeter, different area problem, however, I don't have any rulers, as this kind of equipment is housed in the original room. I do have some geo-strips, so I clip a dozen together and introduce the problem.

Because the strips were connected together, a dynamic situation arose; they could easily be changed to another shape and this provided a different effect to that of using separate rulers or straws.

Having set up the problem I invite the group to offer extension tasks. One student suggests we can work out the diagonal length of each rectangle.

I found this particularly interesting because I had not considered using this as a context for practising Pythagoras. This led to a further idea of graphing the diagonal length against the area of each rectangle.

Another student suggests we make different shapes such as a regular hexagon and regular dodecagon; another suggests we can make some pentominoes...

A problem arose about many different pentominoes could be made from a constant perimeter of 12 units. A further idea was to use two different length of geo-strips, which we defined as unit and half length strips; this provided opportunities for a practical demonstration of calculating the area of, for example, a rectangle measuring $3\frac{1}{2}$ by $2\frac{1}{2}$ units. In terms of curriculum development we discussed a range of potential access points for other problems.

In the following section I discuss curriculum development in relation to why I chose not to use textbooks or schemes; I initially consider why they are widely used as a predominant resource for teaching mathematics.

Exploring some reasons why teachers use textbooks and schemes, and rationalising why I chose not to

The majority of departments I have had contact with rely upon published schemes. These are frequently individualised learning schemes. This is born out by Ofsted [1994:16]:

... most teaching continues to rely on published schemes which provide sequencing in excessive detail and can damage teachers' confidence in their ability to make their own decisions.

and Ernest [1996:7]:

the popular individualized School Mathematics Project (SMP 11-16)... is used in about half of all schools in England and Wales.

Ball [1990:10] refers to Stodolsky's research (1988) into teachers' use of textbooks:

... it is the teacher's responsibility to develop ideas in class. Yet, she reports researchers observe little use of manipulatives or other concrete experiences. Instead, students spend most of their time doing written practice exercises from the textbook.

There are many reasons however why textbooks and schemes are so widely used and these have been identified by Desforges and Cockburn [1987:46] as:

- *... designed by experts... the schemes confer status on the definition of the contents of a mathematics curriculum;*

This is unsurprising, particularly since the inception of the national curriculum (NC) and the plethora of commercially produced schemes which claim to 'cover' NC e.g. the 'New Plus SMP 11-16 Books' are advertised as: "written to improve coverage of the National Curriculum..." (SMP advertising campaign).

- *... serve as a form of curriculum justification to auditors - especially parents;*
- Needing to justify how mathematics is taught continues to be an issue in the political arena, and under constant scrutiny by the media.
- *... schemes present a rich variety of ideas for treating concepts and processes;*
 - *... keep many children productively busy - preferably working at their own level of attainment...*

These final two points relate to the exceptionally demanding and multi-faceted job that teaching is; textbooks provide a structure for learning mathematics which individual teachers can rely upon, thus preserving time for other tasks, such as marking and assessment. This is emphasised by Love and Mason [1995:70]:

Schemes provide a huge amount of security. The teacher does not have to do either large-scale planning (where the curriculum has to be organized into separate topics) or small-scale planning (thinking how to present a particular task).

My concern is how textbooks and individualized schemes monopolise teaching and learning, and create a dependency culture; how they can steadily erode teachers' responsibilities for lesson planning and prevent pupils from being active, responsible learners. I am not however opposed to pupils practising and consolidating knowledge, neither am I in conflict with them 'knowing their tables'. I believe that for people to have the facility to know how to carry out a calculation, and know when to use a specific skill, without needing to stop and revise the skill, is necessary if they are to have fluency and confidence with mathematics. The quotes above have far-reaching implications for teachers to engage with curriculum development. The key issue is to find effective ways of enabling pupils to practise and consolidate knowledge whilst simultaneously maintaining their interest in mathematics²³.

In the following two sub-sections I discuss two main reasons behind why I chose not to use textbooks, these are: the atomisation of learning; and the contextual falsification of skill acquisition.

The atomisation of skills through exercises in text books and schemes

Problem solving challenges learners' perceptions of existing knowledge and, to make progress with a problem, certain subordinate skills need to be used; if such skills are not fully developed then the process of problem solving will serve to consolidate them. Textbooks, on the other hand, frequently offer pupils set

²³ See 'Teaching and Learning' chapter p. 154.

procedures to follow; however, completion of an exercise does not demonstrate a pupil's understanding of the underlying concepts.

These schemes tend to over-emphasize standard written methods at the expense of mental techniques and common-sense calculations. Ofsted [1994:16].

A similar criticism, aimed at Primary textbooks, has been emphasised by Bierhoff [1996:13]:

The more segmented structure of English textbooks is coupled with much repetition and less distinct progression, reflecting an acceptance of less thorough consolidation at each encounter.

Gattegno [1963:54] also challenges the use of textbooks in the way they atomise learning, by providing rapid change:

The syllabus "atomises" mathematics, that is to say it isolates its component parts in order to schematise it. Textbooks atomise it in this way in order to overcome difficulties... However, if it is true that the dialogue with the universe at every level occurs within meaningful situations which mobilise the whole being, this systematic atomisation cannot be justified since it eliminates the true spur to action, the affective challenge. The difficulties seem greater rather than smaller if the mind tackles one point in isolation, at a time when it is attracted by the whole.

By completing an exercise, a pupil may show s/he is capable of following a procedure, but this does not necessarily imply understanding; nor does it mean the learner can transfer the skills to a different context. I describe an example of this in "Proof, Progressiveness and Procrastination"²⁴ where most pupils in a class were unable to recognise the need to apply Pythagoras' Theorem to solve a problem in a slightly different context, despite the fact they had practised the theorem, by working through various exercises, only a few weeks earlier.

²⁴ Mathematics Teaching 155 pp. 40-44.

How can practising routines be justified if at the very point of a need to use a routine, the learner is seemingly clueless?

Ollerton [1996:43].

The falsification of contexts for learning "real-life" skills

To justify mathematics as a practical and necessary subject, schemes frequently use contexts and embellish skills within, so-called, "real-life" examples. The phrase "real-life" is used, in National Curriculum documents for example:

Pupils should be given opportunities to use and apply mathematics in practical tasks, in real-life problems .

DFE [1995:11];

and:

Pupils should be given opportunities to consider how algebra can be used to model real-life situations and solve problems .

[*ibid.*, p. 15].

As such, "real-life" needs defining, particularly when in published schemes its portrayal is typified as follows:

A park is in the shape of a rhombus with one of its angles 60° .

At each corner of the park stands a hot-dog seller.

People in the park go to the nearest hot-dog seller when they want a hot-dog. Draw the boundaries of the sellers' 'territories'.

SMP 11-16 [1985a:52];

and:

18 people spend all day clearing a building site. Altogether they receive £250 and agree to share it out equally.

How much does each person get to the nearest penny?

SMP 11-16 [1985b:15].

Here, the park is not real and neither is the situation of people going to their nearest hot-dog seller. The building site is not real and neither is the notion that a number of people will receive equal pay for carrying out a job. These contexts are an excuse, a gesture towards providing "real-life" reasons for children doing mathematics. My concern is about the pseudo contexts which, in real-life, are

meaningless, together with the kind of messages pupils receive about the nature of mathematics. Furthermore, because adolescents' real-life contexts differ to those of adults, the over-use of textbooks to promote learning becomes counter-productive and unchallenging. In the following section I develop my rationale for devising alternative approaches to using textbooks.

Finding alternative approaches to using individualised schemes, textbooks or repetitive worksheets

From 1986 with the added responsibilities, as head of department, for curriculum development, I decided not to use any published schemes or textbooks in my lessons; similarly, I used repetitive-type worksheets, where pupils practised separate skills in isolation, with diminishing frequency. My decision not to use such resources arose because I came to believe they provided pupils with little, if any challenge, neither did they enable pupils to develop an interest in, or gain excitement in developing mathematical thinking.

Mathematics is not only taught because it is useful. It should also be a source of delight and wonder, offering pupils intellectual excitement and an appreciation of its essential creativity.

NCC [1989:A3].

The conditions that enabled me to make such wide-ranging changes to the way mathematics was taught at Orleton Park were:

- having the support and active involvement of senior and department colleagues;
- having contact with a radical and enthusiastic group of teachers through the ATM-SEG GCSE scheme;
- having greater autonomy over pupil assessment (prior to the imposition of Key Stage tests);
- enjoying the challenge of constructing whole schemes of work and devising individual lesson plans; and,
- a personal desire to make mathematics more exciting, accessible and interesting.

This final point is my focus for the following sub-section.

Planning accessible and interesting tasks

Lesson planning is a principal responsibility which I find an interesting and stimulating aspect of teaching. Devising ways of capturing pupils' imaginations and creating accessible problems to arouse interest is fundamental to my practice; I do not seek to relinquish this responsibility to writers of textbook.

I describe several examples of such approaches throughout this thesis, e.g., 'Four-in-a-line', 'Transformations' and 'Trigonometry'. Below I have collected together others; each starting point represents an active approach:

Starting point	Equipment	Main concepts
Finding all the triangles and quadrilaterals on a 3 by 3 dot grid.	9-pin geoboards and 9-pin dot grid paper.	Classifying triangles and quadrilaterals; area of shapes; angle sizes.
Paper folding.	A6 coloured paper.	Properties of shapes; tessellating shapes; angles; using algebra to define area and perimeter
Constructing 3-d shapes.	ATM MATs, Copydex glue.	Making Platonic and Archimedean solids; Euler's theorem, truncating solids.
Exploring numbers on a 100-square.	OHP and 100-square grid paper.	Mental arithmetic; times tables; algebra.
Finding all the cuboids with a fixed number of cubes.	Multi-link cubes, isometric grid paper.	Dimensions; volume; surface area; prime numbers; constructing formulae.
'Paths' problem leading to Fibonacci sequence.	Cuisenaire rods and square grid paper.	Addition; subtraction; drawing graphs; Golden ratio; algebra.

Folding and cutting isosceles right-angled triangles.	Coloured isosceles right-angled triangles, scissors, Copydex glue, sugar paper.	names and properties of shapes, angle sizes, fractional areas, adding fractions with denominators of 2^n .
Playing 4-in-a square.	Pegs and pegboards.	Co-ordinates; vectors; Pythagoras' theorem.

Having set up the initial task, I suggest extension tasks for pupils to develop the problems further; practise and consolidation arises from the nature of the task. Some of the tasks I have made reference to elsewhere²⁵; a further example is based upon a lesson about Circles²⁶.

Taking risks to open up new possibilities

Taking personal risks

I define risk-taking in terms of the teacher not knowing all the answers or not being able to predict all the mathematics which learners may elicit; whenever I try a new idea, I inevitably take some measure of risk.

Risk-taking is supported by the strength of relationship I have with a class; the more confident I feel, the more risks I am able to take and the more I can rely upon pupils and students to take initiatives and determine the shape of a lesson. In terms of my development, risk-taking is a process of renewal, of focusing energy on teaching concepts in different, more interesting ways compared to how I have taught them in the past.

To illustrate a number of aspects of curriculum development that risk-taking enables, and also to highlight the importance of being part of a professional association, I offer the following anecdote about a discussion at an ATM General

²⁵ Also see Selinger [1994: pp 65-71].

²⁶ This is described in the 'Teaching and Learning' chapter pp. 136-137.

Council weekend in October 1993, with another teacher²⁷. He enthused about having used an idea called Grid Algebra as a starting point for various classes and how different groups had worked at a variety of levels. Whilst I had not used Grid Algebra I had seen a video some years earlier of another teacher, who developed this idea²⁸.

From this minimal knowledge I decide to try out Grid Algebra with a Y8 class; I invited another colleague, who had a non-teaching period, to observe the lesson.

My reasons for this invitation were:

- to take a risk and share this experience with a critical friend²⁹;
- to share a 'new' curriculum idea; and
- to gain feedback.

I draw a 5 by 2 grid on the board with the numbers 1, 2, 3, 4 and 5 in the top row and 2, 4, 6, 8, 10 in the second row, i.e.

1	2	3	4	5
2	4	6	8	10

I take the class through the idea of 'tapping-out' numbers in the boxes and gain responses from the class, who 'chant' answers.

The class were clearly enjoying chanting out answers and this, I felt, was an appropriate way of working with the whole-class.

I draw the same size grid again, only this time write just one of the numbers in the grid, then repeat as before. Having gained further responses from the class I draw a third grid. This time I do not use any numbers, and instead of asking what numbers would be in different places, I ask what operation is required to

²⁷ Aidan Harrington, Head of Mathematics, Cheadle Hulme High School.

²⁸ Dave Hewitt, University of Birmingham.

²⁹ David Pickard, then Deputy Headteacher at Orleton Park School.

move from one place to another³⁰. Next I ask the class to consider how many different routes can be created by moving from one place to another, non-adjacent place on the grid.

As the lesson developed new ideas and pathways emerged. I didn't have a definite map or a discrete lesson plan; instead I worked intuitively in response to the pupils replies. When a lesson flows in this way my confidence grows and my ability to take risks further increases.

The whole class introduction has taken approximately 20 minutes; by now there is a board full of information, diagrams, routes and rules for moving on the grid. I feel I have offered the class sufficient input, and ask pupils to write about the lesson, to describe what they have been doing and what they have understood.

Asking pupils to write about and re-construct what they have been doing provides opportunities to confirm understanding or reveal misunderstandings. Later I asked them to share what they had written with another person. Within one lesson therefore I used different work modes, these were: whole class teaching, individual writing and paired discussion. In terms of curriculum development, Grid Algebra proved a useful vehicle for using different teaching and learning styles. The next two lessons were more structured.

I have written some blank grids on the board with an 'S' (for Start) and 'F' (for Finish). I ask pupils to work out four different routes for 'S' to 'F' on each grid. Once they have written the operations to describe each route I ask them to check their results by choosing a numeric value for 'S' and see if their routes produce the corresponding value for 'F'.

As pupils demonstrate competence and understanding of working with grids with two rows, I suggest they develop the work to 3-row and 4-row grids.

³⁰ A movement was either to the right, the left, below or above a place on the grid.

Differentiated learning outcomes were apparent here, and pupils worked at different levels of complexity within the same problem framework. The problem of calculating unknown starting values, by knowing the finishing values and the routes taken, led most pupils into solving linear equations. A further extension task, about inverse processes³¹ provided pupils with opportunities to develop their thinking further; this contrasts with them practising more of the same type of problems.

Because I wanted the pupils to practise mental calculations I asked them, in the first instance, not to use a calculator; having completed their calculations I suggested they check their work with a calculator.

Jenny and Helen check their results with a calculator and because the calculation they have previously carried out: $(12 + 3 \times 2)$ produces the answer 18 instead of the expected answer of 30 they ask why this has happened and claim their calculators are not working properly.

This created an opportunity for me to work with pupils on concepts about the order of operations, and the difference between the calculator result and the order they have written it.

Taking a risk to try out a new idea enabled me to find an effective context for helping pupils develop, practise and consolidate a range of content skills. As well, I developed my own range of ideas, and I involved a colleague in curriculum development.

Risk-taking for trainee teachers.

For students, taking risks during a school practice can be a daunting prospect. Usually the safe option is to give pupils work from a textbook, particularly if this is a common methodology used by the department. However, rigorous

³¹ Pupils considered inverse processes as follows: the movement from row 1 to row 2, ($\times 2$), was the inverse of moving from row 2 to row 1, ($\div 2$). On a 3-row grid, because the multiplicative factor from row 2 to row 3 is $\times 1\frac{1}{2}$ (one and a half) and the inverse operation is $\div 1\frac{1}{2}$, some pupils recognised that $\div 1\frac{1}{2}$ was the same as multiplying by $\frac{2}{3}$; they subsequently began to develop rules for dividing by fractional amounts.

adherence to a published scheme or text can prevent students from considering alternative approaches and, potentially, creating interesting lesson plans. Yet to take risks by departing too radically from the textbook and changing too many of the usual ways of working can lead to difficulties, often because some pupils do not like change. The types of conditions which pupils may see as a change to their normal way of working may be:

- a different seating arrangement;
- using practical equipment;
- adopting an investigative approach;
- group work and discussion;
- a shift from specific skill acquisition;
- pupils writing on something other than their exercise book;
- pupils writing about mathematics per se.;
- using something other than a text book or a worksheet.

As always there is a balance to be found. If a student is to broaden her/his horizons and experience alternative approaches then risks need to be taken. If such a lesson is successful, the student can build on this, if however the student has a difficult lesson, s/he may tend towards safer lessons in future. For this reason it is important for students to recognise they are indeed taking a risk and, where possible, try to maximise conditions for success.

The following anecdote from May 1995 illustrates a student taking a successful risk. The lesson was taught by Peter, a PGCE student at the University of Manchester. The class teacher had asked him to "revise pie charts" in order to prepare the class for a test. Peter was keen to create an active lesson for the pupils, where they could work on some of their own data rather than responding passively to exercises from a text book. I take up the anecdote half an hour before the lesson, when Peter asks if we can discuss his lesson plans.

Peter has devised a potentially interesting lesson. He plans, initially, to use a standard question-answer technique to remind the class about how to display data in pie-chart form. Next he intends the pupils to construct their own questionnaire on

certain preferences and gain information from exactly twenty other pupils in the class. I mention that mathematics isn't always about dealing with 'easy' numbers and by choosing a data sample size such as 20, pupils would avoid having to deal with other problems, such as rounding up decimal answers.

By asking why he intended to choose 20 as the data sample size, instead of a number which was not a divisor of 360, I potentially made a 'classic' error. Peter explained he wanted to make the numbers easy, so the pupils could draw pie charts based upon the calculation $360 \div 20$; Wigley [1992:4] describes this as 'the path-smoothing' model. I describe my intervention as erroneous because my question led to the student changing his plan immediately prior to the lesson. This can, at worst, lead to a loss of ownership, particularly if the change had not been adequately considered and the lesson does not run smoothly as a consequence. Fortunately, Peter was able to adapt his lesson plan to good effect.

Although the pupils do not usually work in mathematics lessons in this way, they are attending positively to the task Peter has set, and are responding well to the relative freedom of moving around the classroom, to ask each other questions. This enables Peter to further demonstrate his teaching skills through many positive interactions he has with pupils.

Peter had planned a risky lesson which provided pupils with opportunities to actively involve themselves in something other than the usual controlled framework; for the pupils this was a departure from a normal mathematics lesson. The risks Peter had taken clearly paid off; with regard to the mathematics, pupils engaged with skills other than pie charts, which were interpreting decimal answers and rounding off the results they had produced. Furthermore, these skills had emerged in a real context.

The risks Peter took enhanced his teaching skills. His approach offered pupils a challenge, and an interesting lesson which could easily have been taught from the relative safety of the text book or worksheet. Risk-taking was, therefore, a useful catalyst for personal and curriculum development.

Summary

Change and development lies at the heart of this chapter. I have described the key changes I have gone through since first teaching mathematics in a secondary school. I have documented some of my teaching experiences, describing my initial dependency upon textbooks and a methodology based upon teaching skills in fragmented ways.

My first head of department was the foremost change-agent to influence my teaching style, to encourage me to make small shifts in order to teach in more open and investigative ways. Changes in my practice occurred through working collaboratively and by having an investment in the way the department developed. The responsibility heads of department have for staff development is, therefore, considerable.

Similarly, the responsibility my head teacher took for providing me, as a new head of department, with a supportive environment, to develop the mathematics curriculum was equally significant. With his support I was able to help other teachers develop a wider range of teaching styles, and adopt a greater range of strategies for implementing different ways of working.

Recording ideas, sharing lesson plans, creating a central resource area, and teaching and evaluating lessons with colleagues, were all ways in which I engaged with curriculum development.

Joining a professional association was a further significant part of my development. Sharing ideas with teachers from other schools, through the ATM, provided access to a wider variety of ways of working. Having this exposure to a wide range of ideas and teaching styles opened up further opportunities for change; this was fundamental to raising my awareness and developing my pedagogy. Having shared ownership of the ATM-SEG GCSE syllabus, with colleagues in my school, was the glue which bonded our development and our intentions together.

Adopting the Wyndham lower-school modular framework in my new situation was relatively straightforward. Adapting it however to use with Y10 and Y11 classes, to take account of the advent of GCSE and simultaneously consider issues of working with un-setted groups required a deal of curriculum planning. This, in turn, caused me to focus on equality and entitlement. An important aspect, therefore, of my developing pedagogy resulted from becoming more explicit about my rationale. Analysing how I taught forced my awareness of why I taught the way I taught.

Writing story lines, to describe the outline plans for different modules became a highly significant feature of curriculum development. Story lines, described starting points and next tasks; the former under-pinned pupil access and the latter provided appropriate extension ideas. Deciding upon the resources to use, adopting more investigative approaches, and relying less upon textbooks were all aspects of curriculum development upon which I sought to provide pupils with interesting ideas to help develop their mathematical thinking.

I have paid particular attention to issues of fragmentation and how "real-life" mathematics is frequently misconstrued in the contexts in which it is frequently placed, particularly through textbooks and worksheets based upon repetitive exercises. I demonstrate how it is feasible to use problem solving contexts to provide pupils with opportunities to explore the traditional content of the mathematics curriculum to use and apply Ma1 process skills, and simultaneously practise and consolidate Ma2-4, content skills.

The last section on risk-taking is a continuation of curriculum development. I argue that to expose ourselves to a risk, in the context of teaching, we potentially create opportunities for developing our range of strategies and more progressive ways of teaching traditional mathematical concepts. This, in turn is a process of renewal. Recognising curriculum development as a collaborative, professional responsibility in contrast to accepting the status quo, is central to the process of renewal.

Curriculum development, as a catalyst for change, is more than just finding interesting ideas, such as using equipment or trying out problem-solving ideas in the classroom. It requires a more fundamental pedagogical shift, based upon a desire to observe one's own teaching in order to study pupils' learning. A determined approach to curriculum development, to record and share those ideas we wish to offer to others, requires an evaluation of the effectiveness of current working practices. Reflecting upon practice to evaluate the effectiveness of teaching approaches, to promote learning, is a meta-strategy for instigating change.

Chapter 5

Teaching and Learning

Why are we learning all this stuff? Sandra [1995].

Outline

I begin this chapter with my rationale which is based upon the triad of children, teachers and mathematics. I then outline my perceptions of how learning occurs, both constructively and convergently. I describe some specific pedagogic approaches to learning, which are: encouraging pupils to formulate their own questions in order to develop specific skills, and inviting pupils to pursue reasons for learning mathematics.

I discuss the importance of reflective practice for all involved in the process of education, and consider the effect this can have upon everyone's learning and personal development. I explain my theories of learning about the importance of pupils being explicit about what they have learnt, and how they can 'fix' learning by being encouraged to reflect upon and write about their mathematics. I consider my role, as teacher, in this and the types of feedback I offer learners.

I describe specific strategies which I recognised in my teaching: the "3-d's", "Other" and "Mantle of the Expert"; the latter two I attribute to Dorothy Heathcote. Ceding, to pupils, greater responsibility for their learning, to help them be explicit about their learning became a fundamental aim, and I analyse how I sought to achieve this.

I complete the chapter with a discussion about the distinctions I draw between different teaching styles which I describe as "closed" and "open".

Rationale

My rationale for teaching is underpinned by acknowledging the triad: children teachers and mathematics, and the dynamic, changing relationships between them. Re-interpreting Sandra's question, above as:

"Why are you making me learn mathematics?" provides a critical focus on the triad as follows:

- it focuses my wider educational beliefs and intentions;
- it guides the way I organise my teaching; and
- it helps me determine strategies to aid learning.

Learning mathematics is an empowerment; it is each person's right to make informed choices based upon personal interest; it is fostered by curiosity, by a need to know, and a desire to acquire knowledge.

...it is far more important to stimulate a desire to know and provide the opportunity for satisfying that desire than simply to provide information and knowledge ready made¹.

Liebschner [1991: 117].

How I cede my power in order to empower children, through mathematics, is dependent upon how I view mathematics, this is:

- as a way of thinking to assist decision making;
- as a set of tools for solving problems;
- as a vehicle of the mind to provide pleasure resulting from the individual's ownership of solutions to problems.

For children to gain mathematical empowerment, they must be able to understand concepts, procedures and structures, practise skills, and make decisions about how and when to employ them. As teacher, I seek to provide a range of opportunities for them to explore problems and, therefore, actively engage in such tasks.

Obviously it is important to learn some mathematical concepts and practice some procedures so that one is a reasonably skilled

¹ Minute 4 of the Froebel Society principles.

performer (like learning to read musical notation and practice scales...) but it is also important for all students to have an opportunity to solve problems whatever their level of capability.
Romberg [1994:290].

My view of mathematics and the way I perceive it is learnt, determines the range of strategies and shapes my pedagogy:

... the art of teaching has little to do with the traffic of knowledge, its fundamental purpose must be to foster the art of learning.
von Glasersfeld [1992:192].

The mathematical meanings gained by pupils and the investments they make in their learning are, in part, due to the quality of interactions I have with them and they have with each other. What I to say to help an individual develop an understanding is based upon my knowledge of the individual; such knowledge is formed not only as a result of how the individual has responded in the past, but more significantly how s/he is responding in the present.

Recognising the importance of responding to pupils' different needs, particularly in my work with un-setted groups, has led me to construct schemes of work which account for and encourage differentiated learning outcomes. The tasks I construct are intended to provide pupils with puzzles or problems which they will find interest in and, as a consequence, become motivated to want to solve them. Offering pupils opportunities to learn mathematics, based upon what they are currently doing, has implications for the way pupils are grouped, and values the importance of being aware in the moment. Acting upon these awarenesses is a teaching skill I continually seek to develop.

In the following two sections on constructive and convergent learning, the focus is on pupils' learning; underpinning each is the type and quality of relationships and interactions I have with individuals and with groups.

Constructive learning

knowledge is constructed by the individual, not passively received from the environment, and that learning, or 'coming to know', is an adaptive process which tries to make sense of experience... Only students themselves can construct their mathematical knowledge, relative to their own individual experience. In every moment of classroom action, some sort of construal occurs. A teacher needs to influence and interact with this construal.
Jaworski [1992:13].

I define constructive learning as the process of building an understanding of a concept from an elementary starting point. The initial problem must be accessible to open up further pathways for pupils to engage with more complex problems.

... a concept emerges and takes shape in the course of a complex operation aimed at the solution of some problem.

Vygotsky [1986:99].

Choosing a place from which everyone in a class can embark and subsequently develop at different rates is one of my chief responsibilities. I describe this as developing story lines and next tasks². Gattegno also attaches importance to this approach:

All I must do is to present them (the pupils) with a situation so elementary that they all master it from the outset, and so fertile that they will all find a great deal to get out of it. Gattegno [1963:63].

I illustrate this with an anecdote from a Y10 class in January 1992.

I gather the class around the board.

Rearranging the room and therefore where pupils sit, is typical of how I manage my classroom. I choose different furniture arrangements according to the nature of the task I wish pupils to engage with. For the purposes of whole class

² See Curriculum Development chapter pp. 92-93.

teaching I usually move the desks away from the board or screen and ask pupils to sit in two or three arcs. Alternatively I might arrange the desks in rows.

On the square grid board³ I draw a straight line from one intersecting grid point to another and ask if anyone can describe it.

Because I want to provide everyone with an opportunity to provide an answer, the starting question is, by intention, easy and open and, therefore, slightly ambiguous. Over time pupils recognise such questions are not narrow and rhetorical; whilst I intend to drive the focus of the work towards the use of vector notation, I want the pupils to offer ideas about how to describe the line without feeling they must provide me with a specific 'right' answer.

Sonya: "It's straight."
Me: "Good, what else?"
Gary: "It goes from one place to another."
Me: "From where to where?"
Gary: "From there to there."
Me: "Show me."
Gary comes to the board and draws over my line saying:
Gary "From here to here."
Me: "OK, we can use S for start and F for finish, anything else?"
Sarah: "Is it a bit like co-ordinates?"

This aspect of my teaching is part of the 'establishing' process. It is important I lead this part of the lesson in a careful way which does not require pupils to deal with over-complex or abstract thought where ideas might be misconstrued; I want pupils to feel sufficiently comfortable to make simple observations. Whilst Sarah's comment about co-ordinates was not what I had in mind this did not matter; indeed I do not want pupils to enter into a "read my mind" type activity. I felt it was important to work from her comment and, therefore, I focused on her comment of using two numbers, ordered by a horizontal and a vertical

³ An important addition to my room was a revolving board with three different surfaces, plain, square grid and isometric dot.

component, to develop a description of the situation. I therefore had to define the difference between co-ordinates as a point on a grid, and vectors, as a shift from the one point on a grid to another.

I draw various lines on the board and invite pupils to tell me the vector description for each; next I invite pupils to draw lines on the board according to a vector suggested by another pupil.

Although the notion of vectors was not a new concept to the group, not everyone would have the same understanding. It was important, therefore, to ask a variety of pupils to either answer a question or draw a diagram on the board.

Next I ask how can I draw a square using a vector as one of the sides of the square.

I had anticipated at the outset that this was most probably going to be the crux of the lesson and to carry on much further would serve to frustrate pupils who may find drawing the squares difficult, and others who would find it a relatively straightforward task. Recognising such points during a whole class introduction is an important part of my speculative awareness.

I ask for contributions from pupils, either to instruct me how to draw the square, or to draw it themselves on the board. Having achieved an accurate diagram, I set pupils the tasks of drawing their own lines, describing each using vector notation, and drawing a square on each vector.

This process, from asking the initial question to posing the squares-on-vectors problem took approximately 25 minutes. In the remainder of the lesson, I attend to individuals' needs; particularly those less confident pupils who require further input. I am equally aware that other, more confident, pupils will require further challenges.

I help some pupils to draw squares and with others I discuss how they have calculated areas. Some see quite quickly how the values, which define a vector, can be manipulated to work out the area of the square, particularly when one of the

components is 1. I ask if a similar calculation will work if neither component is 1.

It is quite usual for pupils, initially, to hypothesise that the area of the square is calculated by squaring the non-unit component of the vector then adding 1. For this reason I ask them to check what happens to the area of the square when neither component is 1.

Pupils' construction of Pythagoras' theorem will emerge at different rates. By the end of this lesson there will be a range of outcomes; some pupils will be able to verbalise and write down Pythagoras' procedure, others will find drawing 'slanted' squares a sufficient challenge. For the remainder of the module, there will be no further whole class teaching. I work with individuals, pairs and small groups of pupils, who choose to sit together, to construct an understanding of Pythagoras' theorem. This construction occurs by directing pupils to work on the following tasks:

- describing the four vectors that form each square they draw;
- calculating the area of each square;
- connecting the area of the square with the components of the vector;
- writing this connection as a formula;
- writing this formula as a program on a graphic calculator;
- translating this knowledge to right-angled triangles, where the length of the vector is equivalent to the hypotenuse;
- checking the calculation works for non-integer lengths;
- seeing what happens when non-right-angled triangles are used;
- transferring their knowledge to different situations, such as working out the lengths of lines on a 9-pin geoboard;
- carrying out problems in 3-d, such as the length of the 3-d diagonal through different sized cuboids made from multi-link cubes.

This list describes my planned 'story line'; the process is aligned to scaffolding. By the end of the module each pupil will have constructed varying levels of understanding of Pythagoras' theorem.

Convergent Learning

Whereas I define constructive learning as the development of concepts from the initial seed of an idea, I define convergent learning as practising and developing skills within a broader problem solving context. A key aspect of learning specific skills, which are subordinate to the solution of a problem, is to try to create an interest for the learner to engage with the skills:

The best way to create interest in a subject is to render it worth knowing, which means to make the knowledge gained usable in one's thinking beyond the situation in which the learning has occurred. Bruner [1960: 31].

I illustrate this with an anecdote from a lesson with a Y8 class in September 1991. The context for the anecdote is an exploration of the Fibonacci sequence. I had begun the work by introducing the 'Paths' problem, [Ollerton, 1994:65]⁴. Having established the sequence I used it as a context for working on a variety of other problems, for example: exploring what happens when terms in the sequence are divided by the previous term; thus revealing the Golden ratio. In the anecdote below I describe some pupils developing algebraic skills using the same context.

I ask the pupils to work in pairs on the 5-cell Fibonacci problem. This requires each pupil to produce several sets of five numbers, each based upon the Fibonacci procedure, e.g.

3, 2, 5, 7, 12. or 2, 6, 8, 14, 22.

Having created their own 5-cell sequences I ask pupils to give their partner the first and last number of each sequence, who then has to try to calculate the three missing values.

Asking pupils to create their own problems within such a framework, instead of me providing examples for them, was a common approach which the pupils were familiar with. Initially they used trial and improvement to solve the problems.

⁴ In Selinger [1994 pp. 65-68].

I suggest some pupils try to find a way of being able to work out the middle numbers of each sequence other than by guesswork, by looking more closely at the numbers they have produced. After a while some pupils begin to tell me they can work out the middle number by adding the first and the last number together and then dividing this result by 3. Having checked this I ask them to try to write down how this worked and then to try to produce a formulae using f for the first and l for the last number in each cell of five numbers.

More specifically some pupils were developing the skill of using symbols to describe an algebraic procedure; for other pupils this was an opportunity to practise and consolidate existing algebraic skills.

As some pupils complete this task I suggest they try construct sequences using values other than natural numbers, such as fractions, decimals and negative numbers. e.g.

-7, 5, -2, 3, 1.

This meant some pupils used fractions, some used decimals and others used a combination of positive and negative values in order to produce sets of numbers. Then they used the formula: $\frac{1}{3}(f + l)$ to check whether it continued to work for values other than positive integers.

I felt there was an opportunity here for some pupils to develop their algebraic skills further so I suggest they try the same idea using 7 and 9-cell Fibonacci-type sequences.

Over the course of this and the next lesson some pupils were able to make connections between the formula for 5, 7, 9 and 11-cell sequences of values⁵. Pupils were, therefore, learning specific skills through problem solving contexts. Finding such problems, to support an atmosphere where learning particular mathematical skills is enjoyable and interesting is a central responsibility.

⁵ An interesting result occurs revealing a sequence where the numerator oscillates between $f + l$ and $l - f$ and the denominators are: 1, 3, 4, 7... ; which are values in the Lucas sequence.

The fact that a particular syllabus has to be taught, or, at least, that a planned set of concepts and activities has to be covered leads to the sort of 'teacher's dilemma'... how to get pupils to learn for themselves what has been planned for them in advance.

Edwards & Mercer [1987: 130].

Furthermore, looking for opportunities where specific skills can emerge, and upon which, pupils' learning can converge is important as this creates a holistic, inter-connected view of mathematics. This approach enables pupils to make sense of the relevance of mathematics, and is fundamentally different from pupils practising specific, disconnected skills in exercises devoid of context.

Whatever the age or ability of the pupils, the content should not be a collection of largely unconnected items. Instead it should be designed as a structure in which the various parts relate together coherently. It is not just that one item is related to another item, but rather that there is a whole network of relationships.

DES [1985:29].

In both constructive and convergent learning there is a common thread of pupils being encouraged to ask questions and question answers; I develop these issues in the following section.

Asking questions and questioning answers

People are naturally curious. They are born learners. Education can either develop or stifle their inclination to ask why and to learn. Shor [1992:12].

Some adults can quote the formulæ for the area of a circle: "*Pi-r-squared*". However, being able to recall this phrase does not necessarily imply understanding; this lies in their construction of the concept which, in turn, is achieved through experience and by questioning what that experience reveals. Effective learning occurs when we ask questions:

...there is only one instrument in research in order to find answers. One instrument: and that is to raise questions, to ask questions. To question is the instrument. And if you don't question, then don't be astonished that you don't find anything.
(Gattegno) Brown, et.al. [1988:11].

To develop the issue of asking questions, I consider a situation where effective learning occurred as a result of pupils questioning what I had previously told them. The anecdote is from a lesson with a Y9 group in October 1992, and is about two pupils asking: "Can you show us how to do long division?"

The work⁶ was based upon an exploration of fractions as decimals; one task required pupils to produce a two-way division table on 2cm square-dot grid. Using calculators, I asked pupils to do just enough calculations to be able to see patterns and then predict further results. This prompted a variety of discussion, some of which centred on the notion of recurring decimals. My anecdote refers to subsequent interactions between myself and two pupils, Katy and Jane.

Katy and Jane are exploring recurring decimal values of sevenths and I tell them the 'final' 1 on their calculator display of **0.1428571** is the beginning of a recurring sequence.

This is an example of me, unthinkingly, telling pupils something and expecting them to accept the answer. Fortunately not all pupils allow this to go unchecked, and here was a case in point.

They are not satisfied with what I have told them and, after a short while ask me how I knew, or how they could know, that the final 1 on the calculator display is the first digit of a recurring sequence.

I was pleased they showed motivation to want to know, however, I was conscious that anything other than a lengthy explanation might leave them feeling less confident of understanding the process.

⁶ The module is described in Selinger [1994: pp. 69-70].

I attempt to show them how to do a long division, however, my explanations are unclear and they are unable to understand. After further confusion they decide to look at 'easier' fractions with denominators of 5, 10 and 20.

Again my explanations had been inadequate. I had told them a procedure but to them it had been meaningless. Sadly, I believe, many pupils would have given up at this point; however Katy and Jane were persistent.

A few more minutes pass and they return again saying: "Can you show us how to do long division?" I go through further calculations much more carefully this time, and after a few minutes they both say they understand the process. They confirm this by carrying out similar calculations.

Here the pupils' motivation provided me with the impetus to explain more clearly a mechanistic procedure. A significant factor was the pupils' desire that I share my knowledge; they recognised a need, and signalled a preparedness to gain the knowledge they wanted. Such preparedness grew, in part, from the types of problems I offered.

The motivation to master new problems is most likely to spring from having enjoyed the satisfaction of finding solutions to problems in the past. von Glasersfeld [1995:181].

Some moments later, another girl, Naomi, asks if it would be a good idea to extend the grid into the other three quadrants.

Her question opened up other possibilities which I hadn't planned for; this idea, which was entirely of Naomi's volition; it showed her independence of thought and her developing understanding of the inter-connected nature of mathematics. She was, therefore, able to relate concepts in a present context to learning experiences from previous contexts.

...real learning, seems to be more a matter of seeing a question than learning an answer. Sotto [1994:7].

Naomi's question provided the basis for conversations about:

- fractions and decimals in all four quadrants;
- how the gradients of the lines previously drawn in the positive quadrant extend into the third ($-x$, $-y$) quadrant; and
- the effect of dividing a negative number by a negative number.

Within a context of learning about fractions and decimals, pupils engaged not just with the ideas I asked them to think about, but showed initiative to extend their thinking, ask questions and, in turn, gain ownership of the mathematics. Such events endorse my view of mathematics as a multitude of inter-connected ideas, the synthesis of which develops a wider understanding of the world. In the next section I develop the issue of pupils formulating questions.

Pupils formulating questions for themselves and each other

From asking questions and resulting investigation, pupils gain ownership of the mathematics they generate. Jaworski [1994:79].

Construction and ownership of knowledge grows from experience and questioning. Encouraging learners to ask questions as well as answering them is a necessary function of learning. The craft of the teacher to foster questioning, for pupils to make meaning of the ideas they are presented with, is central in a problem solving curriculum, and lies at the heart of understanding mathematics.

In a curriculum that encourages student questioning, - the teacher avoids a unilateral transfer of knowledge. She or he helps students develop their intellectual and emotional powers to examine their learning in school, their everyday experience, and the conditions in society. Empowered students make meaning and act from reflection, instead of memorizing facts and values handed to them. Shor [1992:12].

To encourage pupils to construct questions for themselves and for each other to solve, I look for opportunities to create conditions where questioning is

encouraged and legitimised. In 'Classroom Atmosphere'⁷ I describe an anecdote from a trigonometry lesson; in the same lesson the following event also occurred:

I gather together those pupils who, I assess, recognise that the horizontal and vertical readings, which they have previously worked out for different angles turned by the rotating arm, are the same as the Cos and Sin values from their calculators.

Decisions about which pupils I choose to gather together in order to offer certain types of input, are taken as a result of what pupils have recently been doing. This is different to trying to predetermine months or years in advance, those pupils who are likely to engage in future with different levels of complexities of the mathematics curriculum, as a consequence, placing them in sets.

I ask the pupils to construct their own problems by drawing right angled triangles where the rotating arm is the hypotenuse. I explain a procedure, which is to measure the hypotenuse and one of the acute angles, then use Cos and Sin to calculate the lengths of the other two sides; finally I ask them to measure the actual lengths to check their results. I ask them to pose and answer as many of these problems as they think necessary in order to understand the procedure.

By writing their own trigonometry type questions they practised routines and techniques and took control of their learning. This is different from giving pupils an exercise to complete, where the control rests with the author of the textbook. Issues of responsibility and ownership of the work came to the fore, albeit within a prescribed framework.

Karen and Nathan show me some of their answers; these are written to 7 places of decimal, yet their measurements are to the nearest 0.1cm. They want to know how to match these answers. This provides me with a useful context to remind them how to round answers to a certain number of decimal

⁷ See p. 61.

places. We talk about how to do this and later I show them how to use the *fix mode* on their calculators.

Here the pupils asked questions because they wanted to understand what was happening. In a similar way, Holt [1983:213] describes an experience where children gained ownership of their work by posing questions. This is different to a style where pupils mainly answer the teacher's questions. When pupils are shown trust to carry out tasks in a responsible manner, there exist opportunities for ownership and increased motivation.

Answering questions

Recognising how I answer pupils' questions, and the effect this can have, either to open up or close down future possibilities, is a considerable responsibility. Teaching demands much in-the-moment decision making. How I respond to a similar question asked by different pupils will depend upon both my appraisal of a pupil's current need, and my knowledge that individuals derive different understandings from what I say:

Things can be understood in different ways and the understanding may consist of a variety of things. Sierpiska [1994:4].

Questioning by teachers and learners nurtures understanding. When questions are asked I need to try to decipher what lies behind it in order to decide how I might answer; I make in-the-moment decisions. I differentiate between open types of question such as: "*What will happen if...?*", indicating a preparedness by a pupil to develop and extend their thinking, compared with more closed types of questions such as: "*Is this the right answer?*" or "*What is the answer?*", indicating a need for confirmation. The following anecdote from a Y9 lesson in November 1992 illustrates this:

After a 15 minute, whole-class introduction to Vectors, I pose a problem of finding all the possible vectors on a 9-pin geoboard. I ask the pupils to find ways of knowing they have found *all* the possible solutions. I reiterate this by saying they are not to ask me if their solution is correct, but instead are to provide

evidence to support their answer. After 20 minutes Ian and Matthew proudly tell me they have found them all.

Because Ian and Matthew were not usually the most diligent pupils, I was delighted by the manner in which they had worked on the problem. I asked them to explain why they believed their answer to be correct.

They explain there are just three different starting points: a corner pin, a middle side pin and the pin in the middle of the board; for each of these starting points there are eight points to draw lines to, and because each line can go in two directions, making a total of $3 \times 8 \times 2$ (48) possible vectors.

Although this was not correct, it was nevertheless an excellent response, and I was able to celebrate their achievement. By forcing an explanation I was able to help them establish whether their answer was correct or not.

I continue the theme of asking and answering questions by describing two events where the focus was questioning the value of learning mathematics per se.

Why learn mathematics? (1)

A school year that begins by questioning school could be a remarkably democratic learning experience for the students.

Shor [1992:11].

I frequently encourage learners to question the value of mathematics and the ways they learn; holding discussions on reasons for learning mathematics is something I frequently do, particularly with a new class⁸.

If the students' task is to memorize rules and existing knowledge, without questioning the subject matter or the learning process, their potential for critical thought and action will be restricted.
[*ibid.*, p. 12].

⁸ More recently (March 1997) I organised discussion groups with Undergraduate and Postgraduate students at St Martin's, Lancaster, on the theme: "Why Learn Mathematics".

Placing the onus on learners to discuss the value of learning mathematics can yield surprising outcomes. I illustrate such a sequence of events in the following anecdote with a group of Y8 pupils. The events occurred in October 1994.

In my room I had displayed the ATM Celtic Knotwork⁹ posters alongside some Y8 tessellation work. At the beginning of the lesson Mark points to the posters and asks:

"Can we do some of those line things".

I say we will do some, but not until later in the term.

I responded in this way because we had already done a sizeable amount of shape and space work in the first half of the term and I wanted to focus on other parts of the curriculum; I also needed to remind myself how to do Celtic Knot designs.

Mark nevertheless persists and several times over the following weeks he asks if they will be *"doing them"*; each time I thank him for his interest and promise we will work on *'them'* presently.

There was an issue of motivation here; whilst I chose not to adhere to Mark's requests immediately, it was important to positively acknowledge his enthusiasm.

It is the lesson immediately after half term; before Mark asks again, I explain we are going to do some Celtic Knotwork. I use a whole class approach, going through the basic construction in a methodical, step-by-step manner. After approximately fifteen minutes the class begin to create their own designs. I ask them to continue their designs at home.

Giving a set of instructions to a whole class, whether they are setted or not, can be problematic. It is important to accept some children will understand and follow sets of instructions more easily than others. Others will need more individual attention and this can be provided after the whole class introduction; this can be done either by myself or another pupil¹⁰.

⁹ See Clarkson MT126 pp. 5-8.

¹⁰ Asking one pupil to help another parallels a strategy of Heathcote's named "Mantle of the expert" which I describe in greater detail later in the chapter, pp. 150-152.

At the beginning of the second lesson I ask the class to tell me what they thought the previous days' lesson had been about with regard to learning mathematics. They respond as follows:

"Helping us to carefully follow instructions."

"We had to think about and work out our own designs"

"We were working systematically"

"Helping us to draw accurately"

"We had to pay attention and concentrate."

"We were solving problems."

I was surprised at the nature of some of these responses and, whilst I recognise some pupils fed back the kinds of words and phrases they would have heard me use before, the key principle was about asking them to explain what they thought they were learning, rather than me defining this for them.

It is not enough for students to complete tasks; we must encourage students to reflect on their activity.

Wheatley [1992:535].

Helping pupils achieve self-realisation is central to effective learning. Enabling a 'captive' audience of adolescents to engage with questions about the purpose of learning mathematics is essential for their empowerment. The process of opening up such a debate can be highly illuminating.

To encourage students to talk about what they are doing, and to explain why they are doing it, is the agreed pre-requisite (in being a reflective learner) .. to engender reflective talk requires an attitude of openness and curiosity on the part of the teacher, a will to "listen to the student" von Glasersfeld [1992:443].

Asking pupils to construct meaning, to consider wider reasons and become explicit is an important part of my teaching. In the following section I describe how I offered pupils my explicit reasons of the value of learning mathematics.

Why learn Mathematics? (2)

The role of the educator is not to "fill" the educatee with "knowledge", technical or otherwise. It is rather to attempt to move towards a new way of thinking in both educator and educatee, through the dialogical relationships between both. The flow is in both directions. The best student... is not one who memorizes formulae but one who is aware of the reason for them.
Freire [1974:123].

Having taught approximately three and half thousand adolescents over twenty five years, I recognise that learning Pythagoras' theorem or solving equations may not always be high in pupils' own list of needs or priorities. I am equally conscious that the more I force and coerce pupils the fewer opportunities they will have to take responsibility for, and control of, their learning; furthermore, their learning needs will become subservient to my teaching demands. To empower pupils, to learn Pythagoras or how to solve equations, I must create a classroom atmosphere which juxtaposes a social context with a purposeful, enquiry-based, problem solving pedagogy. This approach supports pupils' questioning the value of learning mathematics; the following anecdote, from October 1995 illustrates this.

I am working with the whole class, describing a starting point for a trigonometry based module when Sandra, a Y11 pupil, asks: "*Why are we learning all this stuff?*".

Sandra was a capable mathematician, yet had an extremely poor self-perception of her mathematical capability; this, in part, was due to personal circumstances. Throughout the previous year she had rarely handed in work which matched her potential. Her attendance was inconsistent and she appeared to be heading for an unclassified grade at GCSE. In many ways she was a classic under-achiever.

Perhaps because of the timing of her question or because, in that moment, I felt her question was inappropriate and unhelpful, particularly as I had often tried to give her support, I didn't respond too positively and brushed her question aside.

I knew however I wasn't satisfied with my response; her question stayed with me and played on my mind. That evening I decided to use her question as the title for a *story-poster*⁸, which I wrote with marker pens on large sheets of sugar paper, and displayed on the classroom wall. My posters read:

"Why Learn Mathematics"

Last week a Y11 girl asked me: "Why are we learning all this stuff?" At the time I didn't give the best answer that I could have done. However the question stayed in my mind and I thought about what answer I could have given to her perfectly reasonable question. This is now my answer:

Most of the mathematics that you will do and learn at school, you will not use in your present or your future adult life. Certain people who may become engineers, for instance, will use a lot of mathematics. However not many students will become engineers. The MOST IMPORTANT SKILLS that you will need to learn and use are:

<i>Smiling</i>	<i>Making observations</i>	<i>Recognising connections</i>
<i>Working carefully</i>	<i>Analysing information</i>	<i>Being creative and imaginative</i>
<i>Solving problems</i>	<i>Using a calculator</i>	<i>Not using a calculator</i>
<i>Learning how to connect ideas together</i>	<i>Being nice to each other</i>	<i>Caring about what you do</i>
<i>Deciding when to work with other people</i>	<i>Making your family and friends proud of you</i>	<i>Deciding when you need to ask a question</i>

<i>Halo polishing</i>	<i>Carrying out a calculation in your head</i>	<i>Deciding when to work on your own</i>
<i>Exploring new avenues</i>		<i>Trust</i>

In mathematics I want to help you to develop all of these skills AND AT THE SAME TIME learn about and use ideas such as: factors vectors constructions trigonometry statistics transformations volume decimals co-ordinates addition angle perimeter sequences 3-D tessellations graphs multiplication shapes & patterns Fibonacci percentages probability area subtraction Pythagoras numbers & patterns matrices division standard form

With hindsight, I recognise my posters contain certain aspects which I would change. For example some of the skills I refer to in the first poster describe more my repartee with pupils and how I encourage them to engage with mathematics.

Nevertheless, the posters were useful for:

- showing pupils I am conscious of the ways I respond;
- being publicly specific about the values I hold as their teacher;
- encouraging pupils to consider their reasons for learning mathematics;
- showing pupils the importance I place upon combining their learning of mathematics with other, seemingly, non-mathematical elements.

The posters provoked useful discussion in all my classes and, significantly, Sandra underwent a positive transformation. Over the following months she produced two outstanding pieces of coursework¹¹. At the time of writing the posters she had been heading towards a GCSE grade U; she eventually gained a grade D, and I trace her massive improvement back to this sequence of events. A door had been opened which she had longed to find yet had lacked the motivation to pass through.

¹¹ On one occasion, for example, she requested if she could continue with one of her mathematics problems during a time when she had been excluded from another lesson.

Being explicit with pupils about my teaching is connected to, and supportive of pupils being explicit about their learning. Encouraging pupils to write about the mathematics they do is to help them fix their learning, I this develop below¹².

Pupils fixing their learning

If reflective thought is, indeed, a forceful motor of mathematical invention, it is only natural to put it to good use in such educational design as is based on the principle of learning by reinvention - by guided invention, which means that the guide should provoke reflective thinking. Freudenthal [1991:100].

Encouraging learners to record and confirm what they have understood, to help them be aware of what they have learnt, is to fix learning. I illustrate this with an anecdote from October 1994 based on work on Circles and π with a Y9 group.

I ask pupils to cut several strips of paper, alternately from the long side, then the short side of an A4 sheet of paper. Having measured the length of each strip, I ask them to sellotape the two ends together and roll each band as accurately as they can into a circle. Then I ask them to measure across the diameter of the circle and record their results¹³.

As pupils complete this task I ask them to look at their results to: (a) find a connection, between circumference and diameter; and (b) round up their results to 2-decimal places.

The approach, of measuring the circumference and diameter of circles, to provide pupils with a concrete experience, is a fairly standard one. By cutting strips from paper and forming circles pupils were creating their own information.

¹² 'Fixing' and 'developing', are words associated with the photographic process, this provides an interesting metaphor for learning.

¹³ A 'skill' I perfected was to speedily stick several strips of sellotape along the edge of pupils' desks. This required me to move, at high speed, around a class. This was an aspect of my practice which pupils acknowledged with obvious pleasure.

The next task I ask them to do once they have completed the measuring and calculating, may be considered less usual:

Towards the end of the lesson, I ask the pupils to write about what they have understood from the lesson; more specifically I ask those pupils who have begun to round-up their answers to describe how the convention of rounding up works.

Asking pupils to be explicit about how to round up, to 2 decimal places, was a development of a 'new' skill for some and consolidation of a familiar routine for others. Similarly, pupils' conception of π as the ratio of circumference to diameter shifts, over time, from a significant conceptual understanding to an obvious one. Encouraging pupils to write about specific skills, as well as the method they have used is commensurate with journal writing:

By asking the students to report in their journals how they solved a problem or approached the study of a topic, they can be encouraged to become introspective of how they do and learn mathematics, and consequently be brought to identify more general heuristics to solve mathematics problems as well as to realize the possibility of alternative approaches to the same learning task. Borasi and Rose [1989:356].

As pupils' skill acquisition shifts from new to an *at homeness*¹⁴, they can decide what to comment upon. The act of pupils writing about their mathematics becomes an inherent aspect of their learning, and encompasses a broader principle of pupils taking responsibility for their learning.

I develop this approach in the first lesson with a new Y7 class by asking them to write about the lesson in such a way that a parent could understand what they had done in their mathematics lesson. Helping pupils find their voice, to create opportunities to explain what they have learnt, what they have found difficult, and how they feel about the ways they learn is an important part of the learning process. As they mature, I help pupils develop their writing skills by asking them

¹⁴ Cockcroft Report [para. 39].

to be more evaluative and analytical about their learning. Below are examples to illustrate this development.

The first comment is by a Y7 pupil in the Autumn term 1992 having completed a module about Factors:

"I think that since I have been in Orleton Park the Maths has seemed more interesting than any other maths I have ever done through all of my schooling. I have enjoyed working with factors because it is something different, and it has been explained much easier to me. I used to find that the maths was boring and it was not explained so clear to me. The teachers would just give me a maths book and let us get on with it. So I have found it a lot, lot better and interesting."

The second was written by Kelly¹⁵, a Y11 pupil, in November 1994 having completed a Trigonometry module:

"I can honestly say I have really enjoyed this project a lot and I have learnt heaps of stuff! The first thing we did with the spinner and the x and y co-ordinates was less interesting and drawing the graph was very time consuming, but all the same it was something new I'd learnt. I was interested to find out that the lines on the graph were known as sound waves and that all sound waves follow the same pattern!
When I went on to the actual 'trigonometry' bit with the right angles etc. I thought it was really clever that just by knowing two things - the hypotenuse and an angle you can work out the lengths of the triangle sides and then the angles. However one thing did seem strange to me is that to work out the lengths of the sides by knowing the angle you need to use the cos or sin button on the calculator. In most other maths projects I have done, all the calculations can be done without a calculator, and the calculator just helps to 'speed' things up. However with these

¹⁵ See Ashworth [1995].

equations, you had to have a calculator, otherwise you couldn't work out the answer...

When we first went out to measure objects we discovered we were using the clinometer wrong, which made the angles incorrect. However we went out a second time and obtained the proper angles, and I was then able to calculate the height of objects. At first this really confused me, as I couldn't work out how the equation went, but after I discovered that I had to work out the hypotenuse each time, it became easier!

Then came the prospect of 3-d shapes which I thought would be really difficult. However, once I knew what to do, it was the same as 2-d shapes.

Finally - the impossible line. I guess that this hasn't really much to do with trigonometry, but it shows that things aren't always what they seem, so I thought I'd include it. I thought that the impossible line is an interesting 'project' of its own, and it really got me thinking¹⁶.

Overall I have really enjoyed this project, learnt a lot and have been interested all the way through."

Pupils' comments offer useful insights; positive ones, of course, provide a welcome boost, less positive or negative comments highlight something I need to attend to. Encouraging pupils to reflect upon their learning is an important factor in their empowerment. The purpose of writing empowers learners to find out, or confirm, what they know.

Nurturing a culture where learners write about their work, and reflect upon what they have understood is one way of fostering effective learning habits. There are parallels here for myself in writing this thesis.

¹⁶ This problem was suggested to Kelly by Mike Askew who was in my classroom in preparation for the BBC TV 'Teaching Today' programme.

By encouraging students to monitor their own learning, teachers can help them achieve greater control over that process.

Schifter and Twomey Fosnot [1993:11].

Initial teacher trainees evaluating their learning

Just as I encourage pupils to write about what they have understood, the same holds true for ITT students. Reflecting upon a session to consider the central issues which arose is an essential aspect of their development as emerging teachers. My response to a student's writing is a further component of their development. I illustrate this with notes written in May 1992 by two students at Keele University.

The first comment, about maths as potentially a social activity, arose from a 'people-maths' problem¹⁷. The problem is about partitioning numbers into successive subgroups, involving one person moving from each group to form a new subgroup. For example with 9 people and a first partition of (5, 4), the following set of partitions occur:

(4, 3, 2) - (3, 2, 1, 3) - (2, 1, 2, 4) - (1, 1, 3, 4) - (4, 2, 3).

This problem can be developed by considering other starting values. Having explored other partitions for 9 people an unsuspecting passer-by was invited to join us and one student wrote:

This week found us in glorious sunshine, so it was decided that the class would be conducted outside. This was a refreshing change and enabled us to impose a little maths magic on the outside world... A passer by was cajoled into making up the numbers. He put up little resistance, but I think the art of surprise and his state of shock was on our side. The result for ten people was not a series of mini-groups (as previously for nine people) just a straightforward 1, 2, 3, 4 subgroups. The passer by was

¹⁷ This idea was described to me by Alan Bloomfield, Cheltenham & Gloucester College of Higher Education, at an ATM General Council weekend.

thanked and 'allowed' to go. We had been given a glimpse of how maths could be fun .

I responded:

You have a lovely writing style Jake. I hope you will be able to think about how you might encourage children to write about their own mathematics as writing about, or verbalising an idea helps us understand what we have done more clearly.

Writing responses to support students' writing encourages explicitness and develops a healthy discourse between teacher and student.

The next comment illustrates a number of issues recognised by one student, Jan, as a result of working on the Irat problem⁸. I have underlined certain words and phrases which I believed significant and used them as a basis for my response.

Jan wrote:

How interesting it became to see that bright yellow paper come alive. Children learn by watching and listening and by following example, but even more by doing, by explaining, by enquiring for themselves. How fascinating number sequences can be. Only by experimenting with and investigating the numbers themselves can young minds find interest in just a list of numbers .To teach by methods of, 'I say, you repeat, you learn' will not stimulate the mind to enquire why. Few children readily accept the fun of numbers, so great is their fear of not understanding. Allow the inquisitive mind to find the patterns and sequences, and the numbers will stay more readily of their own accord in the minds of those looking. All children can do something with numbers besides just counting, but their levels of understanding will be different so perhaps to challenge them to enquire for themselves will encourage even those who say they cannot do maths, to achieve greater than their own 'thought of' potential

Jan indicated her understanding of crucial ideas about teaching and learning; a significant feature was her recognition that pupils' mathematical and social development occur simultaneously. This illustrates the value of students reflecting upon and analysing the processes they engage in.

In the next section I describe the kind of feedback I offered pupils and illustrate this with the comments I wrote to Sandra in response to two pieces of coursework, which I referred to earlier in this chapter [p. 133].

Feedback to pupils

To encourage pupils to write about and be explicit about their learning it is important for me to be explicit about what I had intended them to learn. Writing comments to pupils, therefore, about their work became a feature of my teaching. This grew with the advent of GCSE coursework as a result of the nature of the work pupils produced; I wrote substantial feedback comments to pupils. I came to believe short comments, following a set of ticks and crosses had little impact upon pupils' learning. More significantly, I found the process of writing comments a useful way of entering into a dialogue with pupils. Subsequently I used this style of feedback with all pupils.

Comment 1 in response to Sandra's *Trigonometry* work:

I am aware we had quite a few conversations about your work and feel fairly confident you have been able to achieve a good understanding of the main ideas behind trigonometry. Completing and handing in this piece of work has, I suggest, been an important breakthrough, because this was something that you have found very difficult to organise in the past. I hope this will give you the confidence to recognise you are quite capable of achieving a good standard of work. You have shown you can set up and solve your own problems and work accurately.

Comment 2 in response to Sandra's *Graphs* work:

This piece of work really does show an amazing transformation and is further evidence of your undoubted mathematical ability. You have explored the problems carefully and have been able to recognise the key concepts involved in this project. All of your calculations, that you have set up yourself, are accurate and you have been able to derive the standard formulae for carrying out various types of calculations. You have then been able to extend the idea of gradients into graphs and again here you have had considerable success in seeing how the equation of a line connects with crossing points on both the y and the x-axes. Finally you have developed your graph work into gradients at different points on a curved (quadratic) graph. You can be rightly very proud of the work you have produced here. Well done.

My comments were intended to be diagnostic, confirmatory and, where appropriate, celebratory. By word-processing comments, I developed the quality of my feedback; I also had a record for purposes of reporting achievement.

In the next part of this chapter I describe three strategies which I regularly employed in my teaching; the "3 d's", the "Other" and "Mantle of the Expert".

The 3 d's

In the mathematics department at Orleton Park we frequently used a strategy which we named the 3d's. This strategy describes typical interactions with pupils focusing on what an individual:

- has done;
- is doing; and
- intends to do next.

The 3d's permeated much of my teaching. It was aimed to help pupils become explicit about their learning, to overcome 'stuckness' and decide upon future

steps. I often used the 3d's as a strategy with Y10 and Y11 groups, as a way of beginning certain continuation lessons¹⁸.

I ask the first pupil who sets foot in the room to show me the work they are doing. I am involved in a conversation therefore as the remainder of the class are entering the room. Although my attention is, seemingly, focused on one pupil I am aware of what is happening elsewhere in the room. However, I intend to signal that my attention is with one pupil. Subsequently there grows an awareness, amongst the other pupils, that I am not going to say the lesson has formally begun.

There were important messages about signs and expectations. Usually pupils would settle to their work. This meant I could make a prompt start and simultaneously cede responsibility and greater control to the pupils.

I continue this chapter with two specific strategies which I have adapted from Dorothy Heathcote's practice and, which I frequently use in my teaching. They are the "Other" and "Mantle of the Expert".

"Other"

This is a somewhat fanciful name to give to what might be perceived as just something to deflect the attention of the class.

(Heathcote) Johnson & O'Neill [1984:162].

Many pupils have pre-conceived notions about the names of certain concepts, such as "Algebra" or "Trigonometry", and as a consequence they can convince themselves they will not be able to understand a concept at the outset. To help pupils gain confidence, I look for interesting ways of focusing their attention on the problem or the puzzle at hand, so that, at the outset of a module they engage

¹⁸ I define a continuation lesson as one where I have previously set up the starting point and pupils are working on different problems or are at different stages of the planned story line.

implicitly with concepts. Over time I am more explicit about the vocabulary involved. I describe this process as the "other".

Examples of "others" which I use in my teaching are:

- pegs and pegboards for playing a game of four-in-a-line leading to work on co-ordinates, directed numbers, equations of straight lines and gradient;
- paper folding leading to work on angle, area, perimeter, tessellation;
- pupils being allocated a number leading to work on sequences and functions;
- geo-strips[§] or geo-boards[§] leading to making and classifying shapes, calculating areas, determining symmetries, etc.

When I present an "other" I intend pupils to construct meaning rather than follow a specific procedure; understanding arises from construction of knowledge. I must also ensure they have access to conventional notation and vocabulary. For example when pupils are initially playing the 4-in-a-line game, using a pegboard and pegs, I will shift the focus of attention onto co-ordinates. The results pupils gain from the games provide practice and consolidation as well as opening up extension tasks, i.e. sets of co-ordinates gained can be used for further work on families of graphs[§], which in turn can be a precursor to work on $y = mx + c$ [§].

Mantle of the Expert

This is another strategy of Heathcote's which I recognise in my teaching:

... namely that a person will wear the mantle of their responsibility so that all may see it and recognise it and learn the skills which make it possible for them to be given the gift label 'expert' (Heathcote) [ibid. p. 192].

Pupils are keen to take responsibility for teaching each other; used sensitively, this strategy is not exclusively a one-way strategy where supposedly 'higher' attaining pupils teach their 'lower' attaining peers. I illustrate this with an anecdote from a lesson with a Y9 group in June 1993. We had been looking at various geometric concepts in a module called classifying triangles and quadrilaterals[§].

The initial task involved pupils working with geo-strips to determine conditions under which triangles are formed according to relative lengths of their sides, then classifying the shapes made. As pupils completed this task I demonstrated how to construct a triangle, bisect an angle and draw a perpendicular, with compass, ruler and pencil. In the following lesson I introduced the class to Cabri-géomètre. My aim was for pupils to encounter geometrical concepts using different resources. I take up the anecdote during the sixth lesson of the module.

Katy is a confident mathematician who usually develops her work in depth. She does however have an aversion to computers, despite my attempts to encourage her to work with different programs. She is open and honest about her aversion and when I suggest she use Cabri she responds:

"You know I hate using computers!"

The learner-teacher relationship was such that this was neither meant, nor received, as a negative statement. Katy knew I would respect her wishes to make up her own mind, and I knew she would ask for help when she was ready.

Ian is an MLD-statemented pupil; he struggles to develop ideas in depth. He enjoys using computers and knows how to access different programs. Katy unexpectedly asks if she can use Cabri, and this is an opportune moment to confer an expert mantle upon Ian, so I ask him to help Katy; he is happy to oblige.

Katy accepted Ian's help with good grace and the 'knowing' glance she gave me was an acknowledgement that she had identified another part of my agenda - to build Ian's confidence. This was well meant and equally well received.

Mantle of the expert is a highly effective strategy to serve the following purposes:

- when pupils receive instructions from each other they are more likely to understand the form of language used;
- pupils who give instructions consolidate their own knowledge;
- as teacher I have for a short period of time another 'teacher' in the room, and I can attend to other pupils' needs;
- discussion between pupils is validated and valued.

This strategy weaves many strands of my teaching together, such as: ceding control to offer pupils opportunities to take responsibility; helping pupils fix their learning; it also supports pupils' learning mathematics in un-settled groups.

Ceding the balance of control

To cede control I must carefully shift the balance from telling pupils about mathematics, to enable them to tell me what they have understood.

I expect them to behave as if they're experts, which means I am not going to teach them, they are going to teach us.

Heathcote [1991:VTR 10].

Ceding control is also about shifting the balance of power and responsibility. This is achieved, in part, through the problems I offer, where pupils can organise, control, make decisions about, and gain ownership of their work. The following anecdote from September 1991 illustrates my intention to shift the balance of responsibility. The context is a lesson on constructing equations using Cuisenaire Rods with a Y7 group:

The class are sitting around the board and I ask anyone to explain anything they understand about the word 'equation'.

To cede control I ask questions which have a variety of responses; this is different from asking closed questions where the pupils are expected to produce a specific response; I develop this later in the chapter. How I subsequently deal with and give value to different contributions in order to move discussion on in productive ways is a skill which underpins my craft as a teacher.

I receive various responses then Rachel asks:
"Is it anything to do with the word 'equator'?"

I pursued this question and a discussion developed about the equator splitting the Earth into two equal parts; the Northern and Southern hemispheres. This in turn led to a discussion about the word sphere. There was, therefore, a richness of ideas which transcended a potential narrow view of solving equations.

I introduce the class to Cuisenaire Rods and use them to demonstrate how to write equations such as $r + w = g$. I set the problem of finding as many equations as possible using the following rods:-

1 white, 2 reds, 1 green, and 1 pink.

After a few minutes I notice that Alan, a pupil with a Moderate Learning Difficulty statement, has not settled to the task and is behaving in a mildly disruptive manner. Conscious that this behaviour is likely to disturb others, I suggest he sits on his own for a while.

Whilst I intended to cede control to Alan and offer him with some freedom, his present behaviour prevented me from doing so. I found myself balancing this principle against the rights of other pupils to work undisturbed.

If we give children a completely free choice of activity in the classroom, it will probably mean that the noise of the woodwork table puts an end to the other children's freedom to choose to read a book. Liebschner [1992:64].

Alan has not responded too well to being moved, so I carefully explain the task to him again. This time he settles to his work. After a few minutes he shows me some equations he has created. To my astonishment he has written

$$w + r + \frac{1}{2} r = p.$$

This is a more interesting equation than anything anybody else has produced.

Up to this point the idea of using a half a rod had not been part of the class discussion, so this was Alan's own idea.

The beauty of the Cuisenaire rods is not only that they enable the child to discover, by himself, how to carry out certain operations, but also they enable him to satisfy himself that these operations really work and really describe what happens. Holt [1982:138]¹⁹.

¹⁹ In the revised edition, Holt adds a rider to the above, which I develop in 'Curriculum Development' p. 95.

By balancing the behavioural control with my intention to cede mathematical control, Alan had been empowered to open up new avenues. When I decide to tell a pupil something, or when I ask them to explore something by themselves is part of this balance.

Guiding means striking a delicate balance between the force of teaching and the freedom of learning. Freudenthal [1991:55].

Using a range of strategies and ceding control to the learners, making decisions and recognising how, in the moment, I might effectively respond to a pupil is central to my teaching. In this final section I describe how these approaches were encompassed by developing a more 'open' teaching style, and contrast this with a 'closed' teaching style I formerly used.

Open and closed teaching models

In completion of this chapter I discuss the essential differences between "open" and "closed" teaching models. I do not seek to argue that one is "right" and the other "wrong", instead, I consider my own relative effectiveness as a teacher, having used both approaches.

An open teaching model

In several places in this chapter I have described instances which centre upon "open" approaches to teaching and learning; these are, for example in the sections on: constructive learning, and asking questions and questioning answers²⁰. In the following section I describe a whole-class teaching sequence of events, using a lesson based upon the Irat problem with a Y7 group from November 1994. The anecdote describes a whole class teaching scenario and typifies my open teaching style:

It is the beginning of the lesson and I am holding up a yellow isosceles triangle cut from a piece of A4 paper.

²⁰ See pp. 118-129

I have the attention of the class. There is an air of expectancy as they wait to discover the mystery behind why I am holding a brightly coloured piece of paper in my hand²¹.

I ask *"What is this?"* However, instead of asking for 'hands up' I ask everyone to write down something about the shape. After a minute or so I invite pupils to say what they have written.

Having posed a question I accept several responses, some of which may be obvious or trivial. The key is to constructively build on all responses, including flippant ones, and progressively assemble information about the subject under inspection. Working with the class in this way means everyone has an opportunity to take part. To help a pupil who, I feel, lacks confidence, I may choose to ask him/her to give me the first response.

Sarah: *"It's a piece of paper"* - (some laughter)
 Myself: *"OK what else can anyone say about it?"*
 John: *"It's a yellow piece of paper"* - (more laughter)
 Myself: *"It certainly is - anything else?"*

I choose to confirm such answers as a strategy develop discussion, and because I wish to encourage pupils to offer as many responses as possible.

Alex: *"It's a triangle."*
 Myself: *"Splendid - what do you know about the triangle?"*
 Susan: *"It's got a right angle."*
 Ian: *"It's got two equal sides."*
 Sarah: *"It's half a square."*
 Paul: *"It's equilateral."*
 Myself: *"Hold on, hold on! Only three at once please... I've only got one pair of hands and half a brain!"*

I frequently find myself using this phrase; I use it intentionally to encourage the pupils to laugh and take enjoyment from the situation. I am pleased at the

²¹ This is an example of an "Other"; see p. 144.

plethora of responses and I wish to attend to and analyse each one. I do this by recording their words and phrases on the board.

Myself	<i>"Susan what do you mean by a right angle?"</i>
Susan	<i>"Well it's got a square corner."</i>
Myself	<i>"Meaning - can anyone else answer?"</i>
Ngai Fai	<i>"Its a ninety degree angle."</i>
Myself	<i>"OK so it's a yellow triangle with a ninety degree angle and two equal sides; it's half a square and... it's called equilateral... Why is it called equilateral?"</i>

I am purposefully paying attention to detail and, rather than saying an answer is right or wrong, I want the pupils to establish that 'equilateral' is an incorrect name and to learn what the correct name is.

This open teaching model is a process of scene setting, and building upon pupils' responses. It is also a precursor to exploring a more complex problem. This is to see what happens when an isosceles right angled triangle is successively folded and cut along the resulting line of symmetry²². The open model uses contexts which offer relational understanding. This is typified by Irat problem where pupils have opportunities to connect together and consolidate their knowledge of:

- fractions with denominators of powers of 2^n ;
- equivalence of fractions;
- adding fractions;
- classifying and naming shapes;
- symmetry;
- angle;
- conservation of area;
- exploring a sequence.

²² See Ollerton [1995d:12].

A closed teaching model

I describe a closed model as follows:

- I ask the class a specific question; e.g.

"What do we call a triangle with two sides of equal length?"

Asking the whole class a specific question is not in itself characteristic of a closed teaching model; this depends upon how a lesson develops and the assumptions I make about the responses I receive, and the types of resources I use.

- I invite a pupil who puts a up their hand to provide an answer;

There are tensions here about whether to, seemingly, ignore certain pupils who frequently provide answers, and instead to encourage another pupil, who is usually less forthcoming, to offer an answer.

- If in the event no correct answer is forthcoming I may offer a hint, e.g.

"it begins with 'i'"

My aim, to receive a particular answer, is motivated by a desire to move on to my next question or to the next part of the lesson.

- Having gained a particular answer I must decide what to do next.

What understandings or misconceptions other pupils in the class may have, as a result of gaining an answer from one or two pupils, is something I may consider, and decide upon how to proceed.

This closed model would typically continue as follows:

- pupils practise a set procedure by answering questions from a textbook;

Using a textbook or a scheme, as the central resource for teaching mathematics, is a common feature in classrooms. Teachers are offered many new texts which are promoted as resources for providing a structure for learning mathematics, e.g., *Letts Educational GCSE Mathematics* book is advertised as follows: *The approach could not be clearer, with highlighted key points, numerous worked examples and literally hundreds of illustrations.* (Letts 1996 advertisement)

- for further practice I ask pupils to complete the exercise for homework;

At home - the homework book will be integrally linked to the classbook. It will include topic summaries to refresh the student's memory when working at home, together with homework questions and investigations of graded difficulty. [ibid.]

- I mark pupils' books with ticks and crosses and a brief comment;

The teacher's guide will provide you with answers to the questions in both the classwork and the homework book, as well as guidance on marking. [ibid.]

The textbook is advertised as a complete resource to cater for classwork, homework and marking; as such mathematics is portrayed as a subject that is learnt through example and illustration.

- at the beginning of the next lesson I go through some 'common' errors, using a question and answer technique.

This continues a cycle where particular questions require specific answers.

Sotto [1994:177] describes this question and answer approach as follows:

... this technique has many disadvantages. It tends to make learners follow a teacher passively, and to make them play the 'Guess the Answer the Teacher Wants' game. It also tends to prevent teachers from exploring the answers which learners offer, because they are usually too concerned with getting the answer they want.

Such an approach has also been criticised by Holt [1982:154]:

Practically everything we do in school tends to make children answer-centred. In the first place, right answers pay off. Schools are a kind of temple of worship for "right answers," and the way to get ahead is to lay plenty of them on the altar. In the second place, the chances are good that teachers themselves are answer-centred, certainly in mathematics,...

and by Wood [1988:183]:

The objection, that instruction too often involves attention to procedures and a neglect of conceptual understanding, can be seen as a criticism of many approaches to the teaching of mathematics.

The closed model encourages answer-centredness which, in turn, is epitomised in the way many textbooks present mathematics in artificial, spurious contexts:

A garden gnome 60cm tall stands 200cm away from a fence 120cm tall. Draw a scale diagram showing the gnome, the fence and the ground on the opposite side of the fence from the gnome.

Danuta, whose eyes are 180cm above the ground, moves slowly towards the fence from the left. Mark clearly on your diagram

(a) the position from which she can just see the top of the gnome.

(b) the position from which she first sees the whole gnome.

SMP Book [1985a:56].

Here the context of a garden gnome is intended as a motivator for children to learn mathematics, to focus on specific, 'right' answers. However, the manner in which pupils are expected to suspend their belief and perceive mathematics as a meaningful discipline and powerful set of mind-tools, when they are presented with such resources is demeaning and disempowering. By carrying out exercises from a text books, pupils will practise routines, memorise procedures and demonstrate short-term understanding of specific skills. However, there is little opportunity for them to understand how these skills fit into a broader framework of mathematics; a point reinforced by Daniels and Anghileri [1995: pp 82-83]:

Formally taught written algorithms in arithmetic have little value for pupils who do not understand the procedures that are involved... memorizing procedures often leads to short-term retention and confusion on recall.

HMI also questioned the existence of mathematics as a subject on the curriculum, if learning was limited to practise and computation:

At various points in this document comments are made about the undesirability of overemphasising the practising and testing of skills out of context; the ability to carry out operations is important but there is a danger that skills come to be seen as ends in themselves. If mathematics is only about 'computational skills out of context' it cannot be justified as a subject in the curriculum. DES [1985:11].

I argue that a closed model, based upon textbooks or schemes is less effective, with regard to pupils' learning, than an open model, where the teacher plans his/her own approach and uses ideas gained through experience and from a number of sources in a variety of ways chosen by his/her own volition. Although using textbooks can provide pupils with opportunities to practise specific skills in preparation for a test, there exists a danger that they become the authoritative, all-embracing determinant. Whilst the intention behind both models of teaching is the same i.e. to help children learn mathematics, important differences exist in outcome. Whereas the closed method provides a fragmented, picture of mathematics, the open model offers a holistic, connected view.

Summary

In this chapter I have focused on the children, teachers and mathematics triad and relationships between them. I have described pupils' learning in two key modes: as constructive and convergent and illustrated, through anecdotes, the approaches and problem-solving contexts I use to facilitate learning.

In constructive learning I describe a situation where pupils developed their conception of Pythagoras' theorem, by carrying out a number of specific tasks, e.g. drawing vectors, calculating areas of squares.

In convergent learning I use an anecdote based upon the Fibonacci sequence to illustrate how the sequence can be used as a context for pupils to develop a range of skills, e.g. using symbols to construct formulae and operating with negative numbers. I have described the importance of context, in providing a place where skills exist, and pupils' understanding grows.

Both types of learning involved pupils in problem solving and the development of important process skills e.g. working systematically, making observations.

A central theme of the chapter focuses questioning; of teachers and pupils both asking and answering questions, and of encouraging pupils to question the value of learning mathematics per se. Answering questions is at the heart of teaching; it is the un-scripted nature of classroom interactions, of the different ways I

respond to pupils and students, which makes teaching so interesting and stimulating.

Causing pupils to ask questions to deepen their understanding provides me with challenges to develop my teaching strategies and improve the way I teach.

Further foci in this chapter have been about:

- pupils writing about their mathematics to help them fix what they have learnt;
- students writing reflectively in order to evaluate their learning;
- myself writing to pupils and students to diagnose, confirm and celebrate their achievements.
- specific teaching strategies which I use to enhance the quality of pupils' learning.

I complete this chapter with a discussion about the difference between, and the relative effectiveness of, open and closed models of teaching. I describe particular approaches, detailing typical teacher/pupil interactions, which I build upon to aid learning and show how these are commensurate with an open teaching style, and which I contrast with a closed model of teaching. The motivation to improve my teaching is nurtured by an awareness to question what I do, how I do it and the resources I choose to use. This raises issues of my autonomy which causes me to question the difference between planned, technical rationality and spontaneous, in-the-moment creativity.

Chapter 6

Issues on working with un-setted groups

Without all the colours there would be no rainbow.

Bishop Desmond Tutu [1991]¹.

Outline

This chapter, like the Curriculum Development chapter, is written in two sections: History and Rationale. The historical perspective shows how my principles for working with un-setted groups developed from being a novice teacher at Wyndham School to becoming a head of department at Orleton Park School. I begin with an anecdote from 1976 which had a profound effect upon my concerns about setting. I use documentary evidence, from 1986 onwards, collected from Orleton Park Mathematics' department meetings and documentation, to show how decisions were taken to teach un-setted classes across the 11-16 age range. I complete this section by describing my experiences as an InSET provider.

In the Rationale section I identify and attend to issues related to working with un-setted groups, referring to other teachers' writing, gathered from InSET, about typical concerns and arguments in favour both of setting and mixed-ability. I analyse some of these comments and explore some implications behind them for teachers and children; in particular I focus on:

- Setting or mixed-ability: teacher or pupils' needs?
- entitlement, access and equality of opportunity;
- inclusivity, expectation and equality of opportunity;
- planning to work with individuals' differences;
- a distinction between whole class teaching and keeping a class together;
- open-mindedness about children's potential;
- children are constantly changing: the implications of applying flexible criteria to them.

¹ Taken from a speech for students, teachers and support staff at Peers School, Oxford.

History

Mixed-ability teaching at Wyndham

At Wyndham, in 1973, the mathematics department pupils were taught in mixed-ability groups in their first year². At the end of the year pupils were tested and consequently placed in sets in their 2nd year. This seemed to be a fairly obvious way of operating. However one particular experience in 1976, which I describe below, caused me to question the value of setting.

When I began teaching at Wyndham I was a first year form tutor. In my class were two girls called Anne. Both were hard-working, polite and had cheeky grins. Although neither were strong mathematicians, the effort they put into their work made them a delight to teach. They showed pride and diligence, and through their efforts they raised the overall standard of the class. I had many positive discussions with them.

After the first year I didn't teach them again until they were in the fourth year; both of them were in a set 5 out of 7. I found the transformations they had undergone both alarming and hard to reconcile. They had developed uncaring attitudes and their learning desires had seemingly gone into reverse; both were operating with a lower level of motivation than they had exhibited three years earlier. Any potential they may have had for developing their mathematics was unlikely to be fulfilled in their remaining time at school.

Here, I felt, was evidence of the opportunities and capabilities of two children's mathematical potential being undermined. Accepting the part adolescence played in the changes to their behaviour, I also believe other factors contributed. One was the effect of setting; the anti-learning atmosphere, which was apparent in this low-setted group, had a strong influence.

² I have used both "first year" and "Y7" etc. to describe groups of pupils according to whether I am making a reference pre or post the advent of National Curriculum.

There grew a consensus in the department that teaching in mixed-ability groups could be extended to the 2nd year, and this structure was adopted in 1977. However it was felt that by the 3rd year the gap, between the strongest and weakest mathematicians, was too wide to continue to teach pupils in mixed-ability groups³. Setting therefore, from the 3rd year onwards, became the norm. Some years later, in 1983, I raised the idea of pupils staying in un-setted classes in the 3rd year; however, there was insufficient support within the department and no change occurred.

Transition from settled to un-setted groups at Orleton Park

I joined Orleton Park School in January 1986. The mathematics department policy was to set pupils at the end of 1st year; the main resource was SMP 11-16 booklets. As a head of department I clearly had greater responsibility than as a main scale teacher, and I decided to address the issue of why children need to be placed in sets in order learn mathematics. There also existed encouragement from my head teacher⁴ to put my emerging beliefs and principles into practice. However he insisted that moves to mixed-ability groups had to be for the educational benefit of all the children. Improving all pupils' opportunities, therefore, to learn mathematics was the crucial touchstone. Developing teachers' expertise to teach in mixed-ability groups became a paramount consideration.

As a new head of department, I was aware of the dangers of:

- changing the status quo too rapidly;
- asking colleagues to work in ways which they may not feel comfortable;
- not having a sufficient depth of ideas, nor a wide enough repertoire of strategies for working with mixed-ability groups.

The process was, both by intention and necessity, a slow one. It took five years to carefully consider, and complete the transition of moving from settled to mixed-ability classes throughout the 11-16 age range.

³ In Ollerton [1995b pp35-36] I argue against this notion of the 'gap' being a valid reason for placing pupils into settled teaching groups.

⁴ Peter Hampson offered me, as a new head of department, enormous amounts of support.

Change, in particular under the name of "innovation", is tacitly understood to involve improvement. Rapid changes are known as revolutions, yet revolution need not imply innovation... the innovatory speed depends on the circumstances.

Freudenthal [1991:170].

We made careful moves towards mixed-ability teaching in terms of individual lesson plans, constructing more holistic modules and overall curriculum design. Each stage of development regarding pupils' learning, parental concerns⁵ and teachers' perceptions of a different, more investigative methodology, was discussed and agreed upon; all changes were carefully monitored and re-evaluated at weekly, timetabled, department meetings. Papers were produced prior to and after meetings. These were written up as department bulletins and circulated within the department. The whole process was, therefore, thoroughly discussed, professionally scrutinised and carefully documented. I draw upon some of these bulletins later in this chapter.

Moves to mixed-ability in the 2nd year

A key event occurred in May 1986. A colleague agreed to team-teach his 'top' set 2nd year class with my 'bottom' set over a three week period. The content of the work was 3-d shape and space and we devised a variety of problems for pupils to work on. The outcome of this re-grouping was highly successful, in terms of the work pupils produced. Furthermore, having observed how some 'bottom' set pupils worked by comparison to some of their 'top' set peers, my colleague was unable to justify why pupils had been placed in sets in the first instance. This event proved to be a catalyst to promote discussion, with all teachers in the department, about the implications of not setting for pupils' learning, and upon our teaching. By July 1986 having had two terms to consolidate my position in the department, and to demonstrate the existence of a sufficient range of ideas and resources for teaching in mixed-ability groups, we agreed not to set pupils in their 2nd year.

⁵ With the support of the school Parent Teacher Association, the department held "Maths for Parents" evenings. Here parents took part in workshops and teachers discussed ways we taught mathematics.

Moves to mixed-ability in the 3rd year

Discussions to extend mixed-ability groupings into the 3rd year began in January 1987. I wrote the following department bulletin as a precursor to further change. I have reproduced it in full to show the issues we engaged with.

<u>Mathematics Department</u>	<u>3rd year mixed-ability</u>	<u>25. 1. 87.</u>
<p>I would like us to consider the possibility/desirability of teaching our present 2nd year (Y8) groups as 3rd year (Y9) in un-setted groups. What are the advantages and disadvantages to both learners and teachers of un-setted and settled groups?</p> <p>Does the value of un-setted teaching lie in the flexibility and depth of mathematical activities we organise?</p> <p>Is a 'good' activity one that all students can start on and then pursue either to various levels of difficulty, or explore in a variety of directions?</p> <p>Is a situation that is inherently rich in mathematics a suitable vehicle for 'stretching' all children?</p> <p>Does this in turn challenge their perception of what is attainable, and pre-empt the building of achievement barriers?</p> <p>In my teaching, I endeavour to offer students a wide variety of approaches to mathematics in as many different and active ways I know about, I want to challenge my own and pupils' strengths and weaknesses in ways which allow them to share different insights and to interact and exchange ideas and enthusiasms. I also wish to further the ethic built up in the 1st and 2nd year, of pupils taking more responsibility for their learning.</p>		

Discussions on curriculum ideas and about the different grouping arrangements we could use, continued over the following weeks; after three further meetings, and much debate, we decided to move to 3rd year mixed-ability groups from September 1987. During the academic year 1987-88 no further changes were made. This was to consolidate ways of working with 3rd year mixed-ability groups, and because we did not have a sufficiently broad repertoire of ideas to teach in mixed-ability groups at GCSE level. However, we continued to develop our teaching styles and use a wider range of resources. This laid the foundations for discussions on the viability of working with mixed-ability groups in the upper school. I was also encouraged by the Head of Science⁶ to consider working in mixed-ability with 4th and 5th years classes.

⁶ Tom Johnson played an important part by asking me why teaching children in mixed-ability groups should be any different in the 4th and 5th year than it was in 1st, 2nd and 3rd year classes.

Moves to mixed-ability in the 4th year

The bulletin below, from October 1988, was a precursor to further change. I have reproduced it in full, despite my current concerns which I describe below.

Mathematics Department 4th year mixed-ability 31. 10. 88.

I have become more aware of my wish to move towards mixed-ability groups in the 4th year, particularly since teaching both a 'bottom' and a 'top' set 4th year since September. I feel the learning experience for the 'bottom' set pupils has become demeaning and fear pressures are automatically on my 'top' set students who have a wide range of abilities and therefore eventual achievement. The middle to below 'average' (see below) students in the 'top' set are already failing to keep up with expectations that being in the 'top' set brings and, no doubt, after the initial euphoria of being given 'top' set status they must be wondering why it has all gone wrong. I feel not to have the skills necessary to design a mixed-ability course beyond year 3, because of the ever increasing difficult level of the content that students need to be offered; I am still struggling to overcome this. I do not know, for example how to have trigonometry or standard form type work going on in a classroom for some of the students and have different type of work for other students in the same class. However I seem to be able to hold onto the illusion when I teach these modules to my middle set 5th year group that it is somehow OK, yet I know that I will have 'E' and 'F' grade students from this class and recognise the inappropriateness of these modules for such students. The difficulty is in teaching potential 'D' and 'C' grade students, who must cover these modules to fulfil the necessary content requirements.

Upon re-reading this bulletin I recognised considerable shifts both in my thinking on issues of pupils' entitlement to a common curriculum⁷, and about the kind of vocabulary I used when describing pupils as 'average'. At the time of writing this bulletin, I felt certain concepts were inappropriate for some pupils and, more significantly, that I was able to decide which pupils should have access to which concepts. I no longer believe this to be the case. In any interaction with a group, pupils will gain different levels of understanding; I cannot predict who, nor to what depth understanding will occur. This has become a statement of belief, it is a value which underpins my rationale for not setting.

With hindsight, I took such decisions because I lacked a sufficient range of teaching strategies; this prevented me from offering all pupils potential access to the same curriculum; the key issue is access. However, at that time and even

⁷ I develop issues of entitlement later in this chapter pp. 170-172.

more so today, there are inequitable external pressures created by tiering in the examination system; this produces a framework in which large proportions of pupils are effectively restricted to learning certain aspects of mathematics, according to the level of entry at GCSE. This does not motivate teachers to seek ways of offering all pupils their entitlement to a common core curriculum.

Regarding my use of the word 'average', I implicitly labelled children according to a mis-construed notion of norm-referencing. This was a flawed attempt to measure pupils mathematical potential to place them on an ability scale for my convenience. I was, in effect, trying to simplify highly complex issues.

There is no such thing as the average child, don't presume to cater for it. Jan Mark [1995].

Discussions about how to group 4th year pupils continued into 1989. We analysed our curriculum by deciding which modules were already suitable for use with all groups; we then explored the gaps that existed in the content of the curriculum. As a result we constructed two important criteria for structuring new modules; they were:

- to have a starting point to enable all pupils access to the basic concepts;
- to construct extension tasks to challenge all pupils.

An example of a module which we used with all 4th year groups was the "Max Box" problem: DES (1985: pp 62-64). By February 1989 we had compiled enough modules, which met the above criteria, to teach 4th year pupils in mixed-ability groups for the first two terms.

The bulletin below illustrates the outcome of the next department meeting:

Mathematics Department	4th Year groups 89/90	20. 2. 89.
At the meeting of 3. 2. 89. we again looked at the possibility of having mixed-ability groups in the 4th year. We decided that we would move to this system, with a view to setting, possibly, after two terms. A review will be made at a meeting near Christmas.		

We continued to construct modules and review practice, and by December 1989 had gathered sufficient curriculum ideas to teach 4th year pupils in mixed-ability groups for the remainder of that academic year. We decided, therefore, to keep 4th year pupils in mixed-ability classes.

Moves to mixed-ability in the 5th year

The final stage of teaching 5th year pupils in mixed-ability groups began in March 1990. Although we were sure of our direction I nevertheless wanted colleagues to be clear and explicit about their reasons for choosing not to set. Between the next two meetings, therefore, I asked colleagues to give their reasons for wishing to continue with mixed-ability groups into the 5th year. Below are copies of three teachers' responses.

Teacher X	Teacher Y	Teacher Z
<i>Less tension in teachers and pupils.</i> <i>Dealing with individuals.</i> <i>Natural atmosphere of pupils developing work</i> <i>More student-student discussion.</i> <i>Students developing mathematically as individuals.</i> <i>Constant reinforcement of learning.</i>	<i>Collaborative approach to learning.</i> <i>Non-competitive with peers.</i> <i>Less of an unpleasant atmosphere.</i> <i>Encourages interchange and discussion between teacher-student and student-student.</i>	<i>Non-competitive.</i> <i>No segregation by ability.</i> <i>Students respond to tasks rather than to teachers arbitrary decisions.</i> <i>Learning scheme for the individual, not an individualised learning scheme.</i> <i>Removes labelling and competition.</i>

These comments illustrate the depth of support for moving to mixed-ability groups throughout the 11-16 age range. Statements which refer to: enhancing pupils' learning experiences; creating a healthier 'atmosphere'; and greater social cohesion, both within individual classrooms and across the department as a whole, provided us with a basic pedagogy for the future.

A further benefit, as a result of teaching heterogeneous groups, was to recognise common teaching strategies, which in turn, facilitated a cohesiveness through sharing lessons ideas. This spirit of collaboration continued to enhance our personal and curriculum development.

A key concern, was to provide those pupils, taking the higher tier GCSE, with appropriate opportunities to develop the necessary concepts; with this in mind I wrote "Seeding"⁸. This became a prototype for developing other modules to provide pupils with opportunities to access higher level content⁹.

InSET and teachers' concerns

As the department's reputation for working with mixed-ability classes grew, I began to disseminate its pedagogy and practices more widely, for example at BCME¹⁰ and through InSET; below I describe a 1-day InSET in Shropshire in July 1991.

I began by asking each person to write what they believed the advantages were of settled and mixed-ability groups. During the day I offered various starting points, which the group worked on; having considered each task we discussed the implications for teaching in mixed-ability classes. At the end of the day I asked them to write any further thoughts which had occurred to them; seven teachers permitted me to retain their responses. These comments are re-produced¹¹, integrated and analysed under separate sub-headings throughout the following section on my rationale for teaching in un-settled groups.

⁸ Ollerton [1990:32].

⁹ See Curriculum Development chapter p 93 for a précis of "Seeding".

¹⁰ The British Congress of Mathematics Education has held three conferences: Loughborough in 1991, Leeds in 1993 and Manchester in 1995.

¹¹ A complete set of comments appear in Appendix 4.

Rationale

Alpha children wear grey. They work so much harder than we do, because they're so frightfully clever. I'm really awfully glad I'm a Beta, because I don't work so hard. And then we are so much better than the Gammas and Deltas. Gammas are stupid. They all wear green, and Delta children wear khaki. On no, I don't want to play with Delta children. And Epsilons are still worse. They're too stupid to be able... Huxley [1932:33].

In the Atmosphere chapter¹² I describe an approach to teaching trigonometry, a concept usually taught to pupils aiming for GCSE grades A, B, C and possibly D, i.e. those pupils usually in higher maths sets. Because some concepts are considered too hard for pupils in low sets to understand, they become restricted and lack an equal opportunity to learn certain concepts¹³; this is reinforced when such concepts are excluded from the Foundation level examination syllabus. I do not accept this; with appropriate teaching methods it is possible for all children to understand concepts to different levels. Consequently I do not support a system which labels whole groups of children as 'top' or 'bottom' according to notions of mathematical ability, measured at fixed points by the methodology of testing.

Ensuring equality of opportunity in the classroom will continue to require imaginative solutions, where the best developments in past years of co-operative teaching and learning strategies, effective communication, and a principled mix of whole-class, flexible groupings, pair and individual work, are supported and improved. But central to the improvement of practice must be the recognition that no child's potential is fixed.

Bourne & Moon [1994: 36].

Neither do I support a system where children work blandly in mediocre ways; my philosophy is quite the converse. My rationale, supported by the strategies I

¹² See p. 61.

¹³ See Ollerton [1995a:34-35].

use to provide all pupils with equal opportunities to engage with the National Curriculum. Working with un-setted groups is a catalyst for developing:

- my awareness of the ways children learn mathematics;
- a wider range of teaching strategies to support learning; and,
- more holistic departmental schemes of work.

Setting or mixed-ability: teacher or pupils' needs?

the mixed-ability group teacher needs to be concerned primarily with learning and far less with teaching. His role changes from that of communicator to that of promoter. Lingard [1976:126].

Below are extracts of the comments I gathered from the Shropshire InSET day, to which I referred earlier; I use some of these comments in the remainder of this chapter to discuss issues relating to my rationale for teaching children in un-setted groups. Comments in favour of setting appear in the left-hand column; comments in favour of mixed-ability are in the middle column. Comments in the right-hand column are those made by the teachers at the end of the day.

For purposes of analysis I have coded some of the comments as follows:

- (t) indicates a statement describing a teacher's need;
 (p) indicates a statement describing pupil needs.

For example, two comments from teacher D: *Homogeneous groups easier to teach*, and *Children not labelled*, I have classified as (t) and (p) respectively. Some comments, which could be considered as both, I classified as (t & p).

Advantages of Setting	Advantages of Mixed-Ability	Further thoughts at the end of the day
Teacher A can judge periods of exposition better. (t) can respond more efficiently to individual needs since more individual needs will be <u>group</u> needs (t & p)	able to draw on range of contributions (t) for learning: setting 'right' attitudes to different contributions (p)	exposition? ever? when?

Teacher B <i>The able are not held back. (p)</i> <i>Parents want it.</i>	<i>bottom set - bad news. (t & p)</i>	<i>Nothing against the principle - if anything in favour.</i>
Teacher C <i>This section left blank</i>	<i>weaker gain from input of brighter ones. (p)</i> <i>All can contribute all can gain. (p).</i>	<i>Regular discussions to consider trigger activities; selection of best (ideas)</i>
Teacher D <i>Homogeneous groups easier to teach! (t)</i> <i>able to stretch brighter children in groups of similar ability, (t)</i> <i>able to help less able in smaller groups of similar ability (t & p)</i>	<i>Each child is given the same opportunity. (p)</i> <i>Children not labelled. (p)</i> <i>All children gain from ideas and experiences of other abilities. (p)</i>	<i>Still wondering how I could manage teaching mixed-ability mathematics at 4th/5th year level</i>
Teacher E <i>ability range kept to a minimum (t)</i>	<i>Doesn't categorise pupils (p)</i>	<i>appreciation of "moral" issue.</i>
Teacher F <i>This section left blank</i>	<i>Does not disillusion pupils by putting them in the bottom set in Y7. (p)</i>	<i>Mixed-ability does have a place in the upper school, It's a case of teaching style, getting pupils used to way of working</i>
Teacher G <i>Easier to cover modules leading to public exams (t)</i> <i>Less spread of ability means class can work together on modules. (p)</i> <i>Easier classroom management. (t)</i>	<i>Each pupil can move faster slower as appropriate (p), levelling up (p), equal access (p), not labelling (p),</i>	<i>Potentially good but requires a different mind set</i>

This table shows the totals for each classification:

	Advantages of setting	Advantages of mixed-ability
Teacher needs	10	3
Pupil needs	5	23

Although this is a small illustration there are, nevertheless, implications for the way setting centres on teachers' needs, to create 'lines of best fit' to support the methods they use and match their expectations of pupils' abilities and future achievements, whereas mixed-ability grouping, focuses on pupils' current needs.

Parental concerns

A further element which, in part, dictates how a department chooses to group pupils relates to a perception of parental desire and aspirations for their children. This is exemplified by Teacher B: *"Parents want it"*. What parents want with regard to whether a mathematics department ought to operate in setted or un-setted groups was rarely, in my experience, a widely discussed issue. At Orleton Park school the department ran 'Maths for Parents' evenings; here we positively encouraged debate about the methods we used. On such evenings parents were provided with typical lesson ideas which they worked on during the first part of the evening. Later in the plenary, it was noticeable that parents wished to discuss the mathematics they had done and the interest they had gained from working on the problems; this was in contrast to discussing grouping policies. Parents want enthusiastic teachers who work with their children in interesting ways and build up good working relationships. With such qualities in place they can be confident their children are receiving a sound mathematical education. I contrast this with a conversation about setting, and parental pressure, in 1995, with another head of mathematics.

The head of mathematics favoured setting but was concerned about the expected reaction from two parents as a result of their child not being placed in the top set. However, according to Y7 test scores, the basis used to allocate pupils to sets, the child should have been placed in the third set out of three. Subsequently the decision was taken to have one 'top' set and two, so-called, parallel second sets. However, the child in question was, to all intent and purpose, taught in a set 'three'. This was confirmed by the fact that whenever statemented children from the half year group were integrated into mainstream, they attended this same group.

Issues about what parents want and to what extent schools seek to gather parents views on the issue of setting are problematic. Furthermore, I question how many parents would choose setting if they knew in advance their child was to be placed in a 'bottom' set.

Entitlement, access and equality of opportunity

If we value tolerance, co-operation, initiative, individuality, honesty, the equal worth of all, or social awareness in the sense that all members of society should have some realisation of the problems faced by others, of their feelings, anxieties and attitudes, then we must plan for the attainment of these objectives in our schools. The adoption of mixed ability grouping is thus not merely a structural or organisational change: rather it is a total rethink of our approach to formal education. Davies [1975: 207].

I define entitlement as every child's right to access mathematics, as determined by National Curriculum, and be empowered to:

- engage with the complexities of every-day living in a world increasingly dependent upon technological advances;
- construct and recognise mathematics as a particular form of communication;
- use mathematics as a problem solving tool;
- build confidence to do mathematics and know when and how to carry out every-day calculations.

My responsibility is to provide access to this entitlement without making pre-conceived decisions about the depth of understanding any individual pupils may achieve. Achievement is determined by desires and motivations whatever a pupil's mental faculties may be. Entitlement is strongly linked to equality and is fundamental to my teaching.

The following anecdote from October 1993 illustrates this. The pupils involved, Sandra¹⁴ and Steven, were very different in demeanour and motivation. Sandra's behaviour changed rapidly from extreme happiness to overt surliness; her attendance was inconsistent and she needed frequent counselling; she was capable of achieving a good understanding of the work, however, she had difficulties in sustaining her interest. Steven was highly conscientious and hard-working; once he had acquired a new understanding he was keen to practise it; he had a moderate learning difficulty statement of educational need, which identified

¹⁴ "Why are we learning all this stuff" - 'Teaching and Learning' chapter.

problems with reading and comprehension; he also found it difficult to write about his work. The anecdote refers to lessons in the third week of a module centred upon Pythagoras' theorem.

As Sandra and Steven are working on similar problems I suggest they work together. They agree positively to this suggestion and, in quite excited ways, they keep showing me their work and ask what they might do next.

Although they were being fairly teacher dependent the important aspect was their joint motivation and desire to develop their work.

Steven's determined work habits are positively affecting Sandra, yet it is Sandra who is helping Steven to achieve a better understanding of Pythagoras' theorem.

Creating flexible ways of working, and developing a range of strategies¹⁵, where pupils' strengths are utilised to support other pupils' uncertainties is central to my belief system.

Meanwhile, sitting close to Sandra and Steven, some students are solving problems in three dimensions, whilst others are programming Pythagoras' theorem into a graphic calculators in order to solve a range of different problems.

Opponents of mixed-ability often argue that weaker children, mathematically speaking, are constantly faced with their own failures and feel inferior by comparison to other children, in the same class who are carrying out more complex tasks. I strongly challenge this view; the fact that some pupils carry out comparatively more sophisticated tasks need not act as a demotivator; it is the relationships a teacher constructs that are a key factor in maintaining each child's sense of worth. At issue is the sensitivity and the craft by which the teacher supports pupils' different achievements. It is, I contend, an outcome of setting, where labelling occurs and children in low sets become aware that children in higher set are taught a different curriculum.

¹⁵ Also, see "Mantle of the Expert" - 'Teaching and Learning' chapter pp. 145-147.

When Steven and Sandra handed in their Pythagoras coursework, real success had been achieved; Sandra because she had persevered with a piece of work, when this was something she struggled to do; Steven because he had achieved an understanding of relatively complex concepts. Below is my comment to Steven which I attached to the work he handed in.

You can be extremely proud of this piece of work Steven. You have put a great deal of thought into what you have been working on and I believe that you have been able to gain a sound understanding of the ideas involved. I am particularly impressed at the way you have carefully gone about showing how you calculated the areas of the squares and then later how to use this result to work out the length of the original vector. Both you and Sandra benefited by working together for part of the time on this project and I hope that you will be able to see how important it is to share ideas with other people. I have shown your work to Mr Smethurst and also explained to Mrs Turner and Mr Jones about how well you have done on this module¹⁶.

This comment typified the way I responded to pupils' achievements, and was an alternative to any extrinsic reward system¹⁷. The nature of my comments celebrated achievement, confirmed understanding, pointed out missed opportunities or highlighted misunderstandings. I sought to engage pupils in meaningful ways and, where appropriate, inject humour into the dialogue.

Inclusivity, expectation and equality of opportunity

With respect to making accessible the deep structure of any given discipline, I think the rule holds that any subject can be taught to any child at any age in some form that is both honest and powerful. It is a premise that rests on the fact that more complex abstract ideas can in fact be rendered in an intuitive, operational form that comes within reach of any learner to aid him towards the more abstract idea yet to be mastered. Bruner [1972:122].

¹⁶ Mr Smethurst was a deputy head of school; Mrs Turner and Mr Jones were support teachers for pupils with special educational needs.

¹⁷ See Methodology chapter p. 34.

Inclusivity and expectations are strongly linked to equality of opportunity. The comments: "*Range of contributions*" (Teacher A) and "*all can contribute, all can gain*" (Teacher C), together with the anecdote which follows, demonstrates how these issues are connected.

In all classes there will be a "*range of contributions*", where "*all can contribute, all can gain*". All children, no matter what they have achieved in the past, have contributions to make in the present and the future.

Brightness is a complex quality,... If children are given what corresponds to their make-up, they become bright. If they are denied it, they give up the struggle... we have a share in the formation of less bright pupils, because of our preconceived ideas of how we should carry out our job of teaching.

Gattegno [1963:89].

A significant characteristic of working with pupils in mixed-ability groups is the value the teacher gives to the inevitable wide range of pupil contributions. Deciding who to offer consolidation or extension tasks to according to what pupils have recently done, is significantly different from making decisions about the level of curriculum input children will or will not be able to cope with months or years in advance. Offering some pupils with extension tasks does not preclude others from being provided with an equal opportunity to develop their mathematics commensurate with their present cognitive level.

The more solutions and strategies pupils see and discuss, the more likely they are to develop a real appreciation of mathematics at their own level. Ahmed [1987:17].

Equality of opportunity is embodied by inclusivity and an expectation that all pupils will achieve different levels of understanding of the essential ideas which underpin concepts; building pupil confidence is a central to this. To illustrate this I offer a transcript from a conversation between the Producer of a "Teaching Today"¹⁸ programme and Steven as a Y11 pupil.

¹⁸ BBC TV January 1995: Producer Chris Balmer.

Steven:	<i>I used to have a lot of trouble with maths and different things. I couldn't understand the ways that they did it and why we had to do it, but now I seem to understand more about it.</i>
Producer:	<i>Do you actually enjoy the maths?</i>
Steven:	<i>I enjoy it more than other lessons now. I can get more involved, I don't mind asking anyone.</i>
Producer:	<i>Why do you think you enjoy it more now?</i>
Steven:	<i>I think the reason is because I can join in more, with other things that are going on, I'm not just, just left out or thinking: "How can I do that?"</i>

Steven was clearly aware of the importance of being part of a 'normal' class.

Providing pupils with opportunities to be included, to "join in" (Steven) and not be "left out" (Steven) is an identifiable and significant expectation.

Mathematical content needs to be differentiated to match the abilities of the pupils, but according to the principle quoted from the Cockcroft report, this is achieved at each stage by extensions rather than deletions. DES [1985a:26].

To achieve the aims of pupil inclusivity and curriculum extension, instead of pupil exclusivity and curriculum deletion, my planning objectives are:

- to construct modules within a planned overall curriculum design;
- to decide upon the specific skills pupils would meet within each module;
- to have accessible and interesting starting points for each module;
- to construct a story line of extension tasks for each module;
- to determine useful resources for each module.

This structure contrasts with one where groups of pupils are given different curricula according to the set in which they are placed. Planning work for un-setted groups means new opportunities are offered to all pupils at the beginning of every module¹⁹.

¹⁹ See Ollerton [1995c:35].

Reducing the prevalence and influence of ability stereotyping requires action at a number of levels. First... systems of curriculum and assessment may institutionalise ability stereotypes by setting markedly different goals for different groups of pupils, and offering them very different mathematical experiences. Rather an essentially common structure of curriculum goals and experiences is desirable... What is important is that the system should make these goals and experiences accessible to all pupils. Ruthven [1987:251].

Planning to work with individuals' differences

Recognising and working with individuals' differences is a key factor when developing teaching styles for working with un-settled groups; finding a place from which all pupils can begin to construct knowledge, in different ways and to different depths is fundamental to such planning. My first anecdote in the Teaching and Learning chapter describes some Y10 pupils working on Pythagoras' theorem. To illustrate the diversity of pupils' achievements and the different ways they had developed their work, I use information taken from the end of the first lesson of the module. Pupils had:

- drawn some slanted squares;
- worked out the vectors to describe these squares;
- considered how the four vectors were connected;
- worked out the area of the squares they had drawn;
- looked for and made connections between the initial vectors with the area of the square;
- begun to write a generalisation connecting the vector with the area of the square.

Although the class began this module together, they had spread themselves considerably by the end of the first lesson. Having pupils working on different levels of the same problem need not be a daunting organisational issue, this is

what I expect and plan for. Predicting how far each pupil is likely develop their thinking is an unknown and becomes an unnecessary consideration.

...people rarely just fail to 'learn'; they leave us with the problem of finding out what it is they were learning while they were not learning what we expected them to learn.

Salmon and Bannister [1974:31].

The key planning issue is to consider the range of possible outcomes, and to anticipate the occurrence of different levels of understanding, even with a closely focused task. This is the basis of differentiation.

...structured understanding must take place in individual minds and that the pace at which such understanding develops is almost certain to vary between individuals in any group however it is organised. Bailey & Bridges [1983:51].

I now contrast this with the perception that whole class teaching necessitates "keeping a class together".

A distinction between whole class teaching and keeping a class together

There are a number of perfectly legitimate uses of whole-class activity... These include setting up a class project,...

Wragg [1976:13].

A frequently used argument in support of setting is based upon children's differentiated learning paces, and how, by separating pupils into groups according to ability, teachers are able to provide differentiated tasks and ensure that: "*The able are not held back*" (Teacher B); and the teacher is:

"Able to stretch brighter children in groups of similar ability" (Teacher D).

Both of these comments also relate to issue of the teacher being able to conduct whole-class teaching sessions. After analysing these two comments I attend to

the issue of how whole class teaching does not infer the need to continually keep a class together.

The first comment implies children of 'high ability' are held back by children of 'low ability'; this creates an image of 'high' and 'low' ability children being connected by lengths of elastic, where the former are being pulled back by the latter. In reality all children in a class affect each other and there are a multitude of links between individuals. By extending the premise to a settled group, which still contains children with a range of 'abilities', a logical outcome is that some children are always going to be "*held back*" by others.

The key issue is the skill a teacher employs to enhance the quality of relationships between themselves and children and, children and their mathematics. I consider this issue further using an anecdote from December 1991 with a Y10 class.

I ask five pupils, whom I assess as a result of the work they are currently doing, to collect together around a pair of desks. They have developed their work on a problem about constant perimeter and different areas of rectangular shapes, with some pupils constructing and re-arranging relevant formulæ. Pupils are currently at various stages of writing-up.

This approach of grouping together pupils who are working on similar ideas then offering an extension or a consolidation task to is one I frequently use. In order to decide who would benefit from such a task, I need to be aware of what different pupils are doing. Carrying out and acting upon such on-going, formative assessments underpins my in-the-moment decision making.

I suggest an extension task to some pupils; this is to change the problem from exploring rectangular shapes with constant perimeter and different areas to exploring rectangular shapes with constant area and different perimeters.

This intervention took less than two minutes; it was all I felt necessary to help the pupils develop their thinking further; indeed this was all I intended to offer so that pupils could make their own headway with the problem.

Working in this way, pupils came to expect to be drawn together and offered such minimalistic input; my intention was for pupils to take responsibility for developing their work. Further examples of minimalist, small group inputs are:

- to explore the effect of changing a , b and c upon $y = ax^2 + bx + c$, having worked on linear graphs;
- to explore the effect of changing a and b in $y = asinbx$, having worked on $y = sinx$;
- to explore Pythagoras' theorem in 3-d, having worked on 2-d problems;
- to explore transformations by matrices, having worked on co-ordinate transformations.

A fundamental issue is not needing to separate pupils, on a long term basis, into fixed ability groups in order to 'stretch' them. Differentiation is achieved through the richness of the starting points and supported by planned extension tasks.

There are further key issues about pupils taking responsibility for stretching themselves rather than relying upon myself to do this for them. All children can be stretched or challenged; this is different to only the "more able" being "stretched" whereas the "less able" receive "help" or are provided with a "different syllabus". In terms of equality all children have potential and have an entitlement to be supported and challenged.

Whatever teachers do, and however children are grouped, there will always be a range of cognition by different pupils. Attempts by teachers to minimise the differential, by setting, does not alter this.

... it's impossible not to achieve differentiation in the classroom, no matter what you as a teacher decide to do. Brown [1995: 33];

and

I wonder how many mathematics teachers feel the difficulties of working with children who have a wide range of potential achievements... I do not worry about the 'gap' because I do not expect to keep my class together, nor do I expect all the pupils to reach the same destination. Indeed, to set out to achieve either of these outcomes is to deny the differences which exist.

Ollerton [1995b:35].

At issue is how, through lesson planning, and using a range of strategies and resources, I can help all pupils develop, whatever their so-called ability. Such planning requires deeper consideration beyond a simplistic solution supposedly offered by setting. Accepting children's differences, rather than seeking to minimise the 'ability' range is to offer the "*same opportunity*" to all to pupils to make progress, so none are "*held back*".

In the third column of the table pp. 166-7, listing "further thoughts at the end of the day", there are more interesting comments, particularly: "*Potentially good but requires a different mind set*" (Teacher G). The implication of this statement is commensurate with the development of teachers' practices at Orleton Park: the more experience we had of planning for, and teaching with mixed-ability groups, the wider range of strategies we developed, and the greater our awareness grew about how cater for pupils range of potential achievements. Consequently we developed a '*different mind-set*'.

The real advantage of being asked to teach mathematics to mixed-ability groups is that the teacher needs to rethink not only how he is going to teach mathematics, but also what he is going to teach and how he will organise his teaching. Lingard [1976:124].

My whole class teaching occurred mainly at the introduction stage to a new module. However this did not mean a class was kept together for the whole lesson. By relating the following events, from November 1991, I am conscious of being critical of another teacher's practice and how, as an observer, it is easy to construct alternative ideas. However, I justify the use of this anecdote as it serves to heighten my awareness of the problems of keeping a class together and of the importance of finding extension tasks.

The teacher (Mr A) is head of mathematics at a school where a PGCE student is on teaching practice. Prior to seeing the student teach, Mr A says he wishes to discuss the student with me, and asks me to wait whilst he starts his own class off. He is teaching a 'middle set' 4th year group and the lesson is on indices. He explains how to calculate answers to 'sums' such

as 3^2 , 2^4 etc., then gives the class a worksheet containing five sections, each with twenty sums.

This approach where pupils carry out repetitive tasks to produce right or wrong answers has been questioned by Cockcroft and HMI:

Mathematics lessons in secondary schools are often not about anything. You collect like terms or learn the laws of indices with no perception of why anyone needs to do such things. There is excessive preoccupation with a sequence of skills and quite inadequate opportunity to see the skills emerging from the solution of problems. DES [1982: 462];

and,

...there is a danger that skills come to be seen as ends in themselves. If mathematics is only about 'computational skills out of context' it cannot be justified as a subject in the curriculum. DES [1985a:11].

Mr A stresses that no one is to go beyond section 2, saying he wants the class to stay together so he can explain to them all how to do section 3.

By insisting no one attempted a harder section, he discouraged independence and was unable to cede control for learning to the pupils.

The pupils begin their work and we begin our discussions. Within a few minutes, two pupils tell Mr A they have finished sections 1 and 2 and want to know if they should begin section 3. He reiterates he doesn't want them to do this and instead asks them to do more sums of the same type which he writes on the board.

There were missed opportunities here for supporting differentiated outcomes. Having completed the initial task; a more economic use of the pupils' time may have been to work on a task aimed at deepening their understanding of indices, rather than doing more of the same. This notion of pupils progressing more or less at the same 'pace' ignores individual differences; consequently such differences are not taken into account in lesson planning.

Differences, as I have previously argued, exist in all groups, settled or un-setted, so the principles of acknowledging individual differences, of not attempting to keep a class together are omnipresent. With un-setted groups the teacher is obliged to take these differences into account and, on this point I reject Hart's conclusion [1979:210]:

The mathematics must be matched to each individual and teaching an un-setted class as an entity is therefore unprofitable.

The issue of recognising and working with individual differences, of providing extension or consolidation tasks, as a result of how pupils are responding at a particular time, is substantially different to 'matching', in advance, different tasks to individual children according to the teacher's pre-conceived idea of what each child's needs are likely to be.

Furthermore, Hart's subsequent statement contradicts her earlier one:

Indeed, in a class where the children are supposedly matched on attainment there will still be a wide range of individual needs.
[*ibid.*, p. 210].

Whilst I agree it is unprofitable to attempt to continually 'whole-teach' any class as an entity, this does not mean an un-setted class cannot be given whole-class instruction at certain times, specifically, as I have argued, at the beginning of a topic. Hart's contradiction lies in the fact that any settled group is heterogeneous; it will contain a mix of children with different interests, enthusiasms and every other factor which affects learning. As such the notion that no mixed-ability class can be taught as an entity implies that no class, whatsoever can be taught as an entity. This is patently absurd.

This issue that any so-called homogeneous class contains a heterogeneous mix of pupils is raised by Bailey and Bridges [1983:49].

... no allegedly homogeneous group can actually be homogeneous in all respects affecting learning within the group. Individuals will differ in their ability to listen, concentrate and understand; they will vary in their critical thinking, their sensitivity to

criticism... No group however settled or streamed can be without significant variety.

In a carefully designed curriculum with appropriate teacher support, all pupils can construct and develop tasks according to their needs. An illustration of this is when pupils are given the responsibility to practise a particular skill a sufficient number of times until they feel confident they have mastered the skill²⁰.

The remainder of the anecdote raises further important issues about pupil responsibility and teacher ~~lusts~~.

We return to our conversation, but are soon interrupted by the same pupils; they have now completed the extra sums written on the board. Mr A is about to set them some more sums and I ask if he has any objection if I work with them for a few moments; he takes up my offer. I ask the pupils if they know what the 'power' key on their scientific calculators does. They both confirm this by telling me about squaring and cubing, so I ask them if they know what the power of a $1/2$ key does. They both say they don't so I set them the task of exploring this. I suggest they key in different starting numbers, write down each of their answers, and discuss what happens. I return to my conversation with Mr A.

Later the pupils show me their work; they have collected a lot of information but un-systematically. I suggest they try to impose order on their work and they return to their task.

It is the end of the lesson and they show me how far they have progressed. They are able to predict an approximate result for 10 to the power $1/2$ and confirm this by carrying out the calculation. At this point Mr A realises what the task is they have been working on and says: "Try 16 ... try 25"

²⁰ See Teaching and Learning chapter pp. 124-126.

This intervention ran contrary to my intentions. By providing the pupils with the key to solving the problem, they were prevented from personal discovery; this removed spontaneity and took ownership away from them. This exemplifies 'teacher lust'; a characteristic observed by Boole in Tahta [1972:11]:

The teacher has a desire to make those under him conform themselves to his ideals... the teacher wants to regulate the actions, conduct and thoughts of other people... to arrest the spontaneous actions of other minds, to an extent which ultimately defeats its own ends by making the pupils too feeble and automatic to carry on his teaching into the future with any vigour... he acquires a sheer automatic lust for telling other people "to don't"... in a way that destroys their power even to learn at the time what he is trying to teach them..

Although the pupils had been keen and willing to explore the situation and work together productively, the teacher was more concerned about them producing the 'right' answer as speedily as possible, rather than working something out for themselves, and gaining an understanding of the situation.

For learners to gain appropriate levels of understanding I must be aware of my responsibilities, of the implications of creating restrictions, of providing limitations, or of having too high or too low expectations. The underlying strength of teaching in un-settled groups is being able to have, and to transmit, an open-mindedness of each child's potential. I develop this in the following section.

Open-mindedness about children's potential

Our teacher sets us a problem and then we can develop it as we want and take it as far as we want. Kelly a Y11 pupil²¹.

A central reason for teaching in un-settled groups is because it is unfeasible to make accurate, long-term predictions about pupils' future achievements.

²¹ Teaching Today BBC TV 1995.

Comments by teacher E: *"Doesn't categorise pupils"*, and teacher G: *Each pupil can move faster or slower as appropriate* are synonymous with being open-minded about pupils potential. The following is taken from the same BBC TV programme, here Steven is showing the interviewer his folder of work and describing specific events from a past module.

That's the one where I felt quite, very brave. But then, when I showed it to Sir and he says, well he says, that's OK, he says, but it's going a bit too complicated for this part, where you are at the moment.

This is an illustration of making an in-the-moment decision. Here I believed a certain avenue of exploration, with regard to what Steven was currently doing, would be unprofitable. This contrasts sharply with making long term decisions about whole areas of the curriculum to which pupils ought or ought not be given opportunities to access, according to which set they are placed in. Not pre-determining long-term outcomes forces my awareness to help pupils develop in the present. Whilst I can describe ways pupils have engaged with mathematics in the past, I cannot predict long term outcomes and there will be surprises; indeed being prepared for the unexpected is important. This was adequately summed up by a Manchester University PGCE student who said: *Expect the unexpected*²².

I previously mentioned some surprises such as the Pythagoras work Sandra and Steven did, and her sudden improvement after seemingly lying mathematically dormant for several months. Had I pre-determined they would not be capable of attaining certain understandings of concepts traditionally taught to pupils in 'higher' sets, then their achievements would not have been realised. The act of pre-determining achievement, particularly a pupil's lower achievement by comparison with others, is aligned to Donaldson's description of 'definers':

If the child is defined as a failure he will almost certainly fail, at any rate in the things which the definers (people who define others) value; and perhaps later he will hit out very hard against those who so defined him. So we know at least something to avoid. But we must contrive to avoid it not merely at the surface

²² Ollerton [1995d:12].

of our behaviour. If we do not genuinely respect and value the children, I am afraid they will come to know.

Donaldson [1978: 114].

Rather than trying to define what any child might achieve, and arrive at such conclusions as a result of applying specific criteria, I believe it is important to recognise and celebrate a wider range of qualities and achievements. Keeping an open mind about each child's potential is consistent with a belief that everyone can achieve; my responsibility is to provide supportive conditions upon which this is possible.

Children are constantly changing: the implications of applying flexible criteria to them

It is obviously important that any grouping of pupils is flexible, to allow for different rates of progress and different profiles of attainment across different attainment targets as well as possible errors in assessment. Thus dividing pupils into rigid sets or groups according to their previous results is likely to lead to considerable inequities. Askew et al., [1993:27].

To draw lines, above which some children are deemed to have a certain ability and below which other children have a lesser ability and then to use such information to create setted groups, is a major reasons for children's underachievement. To seek a standard interpretation of 'ability', based upon a measuring device, usually a test, which, were it to be applied on a different day, would render a different set of results, is implausible.

Children's attainment, interests, workrates and achievements change and develop at different speeds. As such it is impossible to objectify, construct and apply fixed criteria which, under scrutiny, maintain integrity and equity, and then use such criteria to place pupils in setted groups.

Recognising pupils' present achievements, rather than seeking to pre-determine their future achievements, underpins my rationale for not setting, categorising or separating pupils into ability groups. The propensity to classify pupils and place them in sets is intrinsically a systemic, teacher-centred activity, which creates the conditions for under-achievement. At the centre of the learning process are children who have entitlements, rights, hopes and aspirations; as such education must be implicitly, explicitly and intrinsically child-centered.

Don't forget that the child is a living thing, with thoughts and beliefs, hopes and choices, feelings and wishes: helping the child with these must be what education is about for there is nothing else to educate. Gillard [1992:92].

Summary

In this chapter I have described key factors which influenced me and how:

- I built my values for working with pupils in un-setted groups;
- as a head of department becoming more pro-active in creating the conditions for teaching pupils in un-setted groups,
- devising a range of strategies to implement these values;
- the process of change was evolutionary, in keeping with principles of ownership and, for reasons of staff development, how this was a shared, negotiated process.
- seeking to translate strategies and methodologies, via InSET, in support of principles of mixed-ability, to teachers in other schools.

I have explained my rationale for teaching pupils in un-setted classes, and discussed issues of equality of opportunity which are fundamental to my pedagogy; these are:

- pupils accessing their entitlement to a common curriculum;
- inclusivity and expectation;
- recognising children's potential;
- children's changing achievements in contrast to having a fixed ability.

Intrinsic to and in support of my principles for not setting, is the development of teaching strategies to work with childrens' differential learning outcomes. I distinguish between notions of whole class teaching and keeping a class together, and offer frameworks to enable all pupils opportunities to learn at a pace commensurate with their current desires, motivations and interests.

I complete the chapter by describing the importance I place upon being open-minded about children's potential achievements and the dangers of assigning pupils to certain sets.

My work as a classroom teacher for twenty five years has caused me to be firmly committed to the principle of teaching children in groups where no pre-conceived notions of ability are made. Teaching children in un-setted groups is fundamental to providing them with access and entitlement to, and equality of opportunity for learning mathematics.

Chapter 7 Conclusion

In the Methodology chapter I wrote about the inter-connectedness of issues. For my conclusion I continue and develop this in order to show how the issues are connected together:

With regard to *equality* I believe *all* children have common, basic *entitlements* to be provided with *opportunities* to *access* the same mathematical concepts.

To support these values I offer all pupils common, *simple starting points* at the beginning of each *module* of work. Subsequently, because different children will develop ideas to different levels of complexity, I must take *differentiated* outcomes into account. To accommodate this I create *story lines* for each module; this involves devising *extension* or '*next*' tasks. Through the construction of modules, I seek to devise a *holistic*, approach to the curriculum, where concepts emerge and skills merge. This *inter-connected* approach describes a methodology which contrasts to one where skills are primarily learnt as separate entities in *fragmented* ways.

Within each story line framework I expect to have a range of types of *teacher interventions* with pupils; this will include the different ways I may *answer* the same question from different pupils.

A significant responsibility for me as teacher is to be involved with *curriculum development*, to look for *problem solving* approaches, to provide *challenges*, to stimulate pupils to *explore* situations and, as a result, *construct* their understanding of mathematics. Engaging with curriculum

development, to enhance my repertoire of teaching ideas and to develop and use a range of strategies and *resources* to aid learning involves *taking risks*. This in turn will depend upon *my confidence* at employing certain approaches and the quality of the *relationships* I have with individuals in a class.

How far an individual is able to, or chooses to, develop a task is dependent, in part, upon their interest; this relates to how much *responsibility* and how much *ownership* pupils can take for their work. For pupils to have opportunities to take responsibility I must find ways of releasing my control in order to *cede control* to the pupils; similarly I must help create an environment where pupils can own the work. This means paying attention to the *atmosphere* that I am partly responsible for creating in my classroom. This atmosphere must be conducive for pupils to want to learn.

Increasing *pupils' confidence* to learn mathematics, and aiming to enhance the quality of the relationships that exist between myself and pupils, is central to the construction of classroom atmosphere. Nurturing a *positive environment*, therefore, of *care* and *trust* is important if I am to cede the ultimate responsibility for learning to learners. Whilst I, obviously, cannot do the learning for the pupils, I can help them *evaluate* and *fix* what they have learnt and I can provide pupils with *feedback*. Through my feedback I aim to analyse and, wherever possible, celebrate the work pupils have done.

An important part of my classroom atmosphere is to encourage pupils to consider *why* they are learning and, therefore, to *question* the intrinsic value of learning

mathematics per se. This, I believe, is fundamental to effective, long-term learning.

Being aware of and employing a range of *strategies* and deciding when it is appropriate to implement them, with either a whole class, or with an individual, underpins my teaching. For example, encouraging one pupil to take on an *expert mantle* and teach another pupil serves the purpose of having more than one 'teacher' in the room and at the same time deepens the 'expert's' knowledge.

A key aspect of my classroom environment and a further strategy upon which I set out to elevate pupil achievement is through *display* work. Displays not only enhance the appearance of the classroom, as a place of learning, they are also useful as a means of causing pupils to summarise certain aspects of their learning.

Setting out with *expectations* that all pupils have the potential to understand of all of the mathematics curriculum, as defined by the National Curriculum, means paying attention to *inclusivity*. The story line process I referred to earlier of providing all pupils with common starting points is intrinsic to my principle of inclusivity. Consequently I do not need to attempt to *pre-determine* the outcomes of individuals' learning; indeed I cannot predict with accuracy what any pupil might achieve, nor can I define any pupil's rate or depth of learning. I can aim to act *spectively, in-the-moment*, and respond to pupils' current mathematical needs, as I perceive them.

Not pre-determining pupils' future achievements means I have no need to separate pupils by any notion of measured ability, nor offer whole groups of pupils different curricula as a result of having separated them into 'fixed' ability groups.

The implication of this, for me, is to teach children in *mixed-ability* or *un-settled* groups.

Finding ways of not needing to separate pupils into different sets has become a fundamental principle; based upon this is my desire to seek ways of constructing equality of opportunity in my mathematics classrooms for those I am responsible to teach.

In my introduction I quoted from Plowden [para. 505]. I offer the same quotation again, here in my conclusion, in order to draw comparisons between the issues Plowden raised and my values described above.

A school is not merely a teaching shop, it must transmit values and attitudes. It is a community in which children learn to live first and foremost as children and not as future adults... The school sets out deliberately to devise the right environment for children to allow them to be themselves and to develop in the way and at the pace appropriate to them. It tries to equalise opportunities and to compensate for handicaps. It lays special stress on individual discovery, on first hand experience and on opportunities for creative work. It insists that knowledge does not fall neatly into separate compartments and that work and play are not opposites but complementary. A child brought up in such an atmosphere at all stages of his education has some hope of becoming a balanced and mature adult and of being able to live in, to contribute to, and to look critically at the society of which he forms a part.

My intention in writing this thesis has been to describe and evaluate the experiences and influences which have guided me in constructing my pedagogy. By linking my practice to my developing theories, through the processes of reflection, analysis and synthesis, and, by becoming explicit about my strategies and principles, I have come recognise the existence of a two-way process. This is how my values impact upon and shape my practice

and how, in retrospect, the explicit recognition of my practice can further shape my values.

In hindsight, writing this thesis has acted as a vehicle for raising my awareness of the numerous, interconnected issues involved in teaching and learning. Keeping issues of equality, access and pupils' entitlement to mathematics, in clear focus, I must continue to look for ways of improving my current practices as a teacher. I must also be capable of adapting and adopting, developing and changing the methods I use as my awareness grows.

My fundamental values are defined by over-riding responsibilities; these are to construct ways of providing all pupils with an equality of opportunity to gain access to learn mathematical skills and concepts.

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Appendix 1a

Page numbers and dates of those anecdotes used in chapters 3, 4, 5 and 6:

A.	pp. 61-62	16-09-91	Q.		23-09-92
B.	pp. 128-129	16-09-91	R.		23-09-92
C.	pp. 129-130	16-09-91	S.	pp. 125-126	15-10-92
D.		18-09-91	T.		07-03-92
E.	pp. 179-180	19-09-91	U.		26-10-92
F.	p. 146	25-09-91	V.	pp. 131-132	07-11-92
G.	p. 71	26-09-91	W.		13-09-93
H.	pp. 122-123	30-09-91	X.	p. 171	13-09-93
I.	pp. 68-70	02-10-91	Y.	pp. 150-151	04-10-93
J.		19-10-91	Z.		28-10-93
K.	p. 177	11-12-91	aa.		04-11-93
L.	pp. 118-121	29-01-92	bb.	pp. 107-109	08-12-93
M.		07-02-92	cc.		11-01-94
N.		12-09-92	dd.		11-01-94
O.		12-09-92	ee.		14-01-94
P.		16-09-92			

Appendix 1b

List of issues (strategies and principles):

1. entitlement (p)	23. creating a positive atmosphere (p & s)
2. trust (p)	24. holism (p)
3. confidence (p)	25. "other" (s)
4. simple starting points (s)	26. choice, independence and autonomy (p)
5. justifying learning mathematics (p)	27. developing 'next' tasks and 'story' lines (s)
6. recognising potential	28. pupil responsibility and ownership (p)
7. ownership (p)	29. the 3-d's (s)
8. using problem solving instead of text books (s)	30. pupils reflecting upon and writing about their learning (p & s)
9. teacher responsibility	31. students being creative and imaginative (p)
10. teacher reflecting upon practice (p & s)	32. returning to the known (s)
11. avoiding fragmentation (s)	33. seeking to provide interesting tasks (p & s)
12. teacher intervention - telling and not telling (s)	34. mantle of the expert (s)
13. the unexpected (p)	35. pupils setting own practise examples (p & s)
14. balance of control (s)	36. not "keeping a class together" (p)
15. ceding control (p)	37. reasons for learning (p)
16. using process skills in context (s)	38. self assessment in coursework (s)
17. working on the 'basics' (p)	39. what I think mathematics is (p)
18. team teaching to support curriculum development (s)	40. what pupils think mathematics is (p)
19. teacher honesty (p)	
20. differentiation by outcome and task (p & s)	
21. range of resources (s)	
22. teaching specific skills problem solving (p & s)	

Appendix 1c

Two-way table to identify the issues 1 to 20 and the anecdotes within which they occurred.

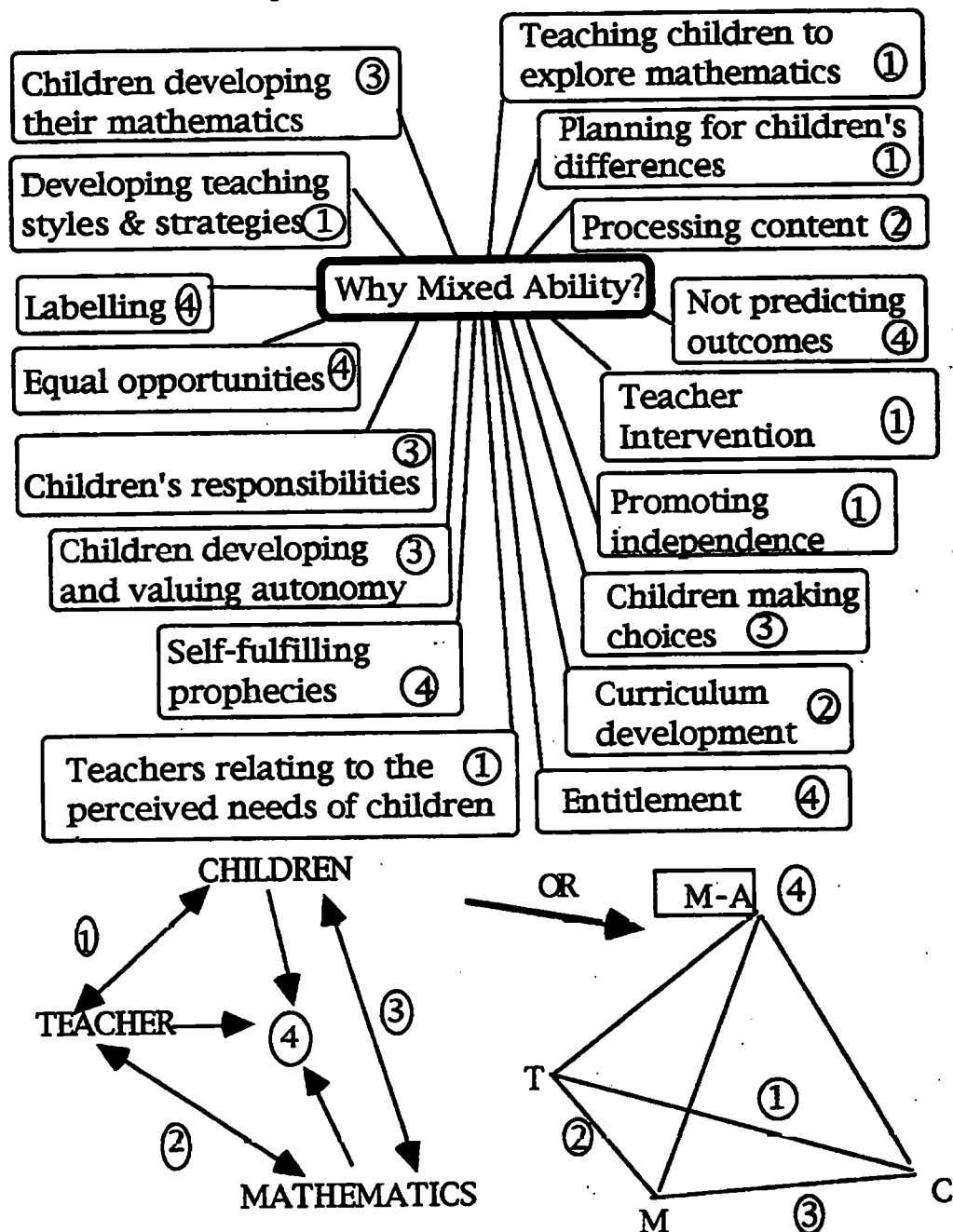
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	✓	✓	✓																	
B							✓	✓			✓	✓								
C												✓								
D													✓	✓						
E											✓									
F				✓											✓	✓	✓	✓	✓	✓
G							✓	✓												
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cc																				
dd									✓											
ee				✓								✓					✓			✓

Two-way table to identify the issues 21 to 40 and the anecdotes within which they occurred.

[illegible]

Appendix 2

Developing the Teachers, Mathematics and Children triad into a tetrahedron to include issues of: Why mixed-ability?



Appendix 3a

The ATM-SEG GCSE syllabus

I attended the first ATM-SEG GCSE meeting on 1-3-86 with two of the syllabus writers, Alan Eales and Noel Fowler. This meeting was to ascertain how much interest there was in the proposed syllabus, and for ATM-SEG representatives to consider which departments might pilot this innovative GCSE.

This syllabus was a development arising from an ATM Assessment Working Group. The origins of the syllabus had grown from paragraph 6.3 in the 1985 National Criteria for GCSE: *Each scheme will contain at least one end-of-course written examination paper. One or more written examination papers should normally account for at least 50 per cent of the assessment.* DES [1985:5].

Defining the word "normally" became the access route for the creation of the syllabus; the ATM syllabus writers successfully argued that modes of assessment at the margins, based upon 100 per cent coursework, was encompassed by a definition which would fall within a normal range. Seven schools, from Cumbria to Avon agreed to operate the syllabus: Stainburn School in Workington, Solway School in Silloth, Reddish Vale School in Stockport, Orleton Park School in Telford, Peers School and Cheney School both in Oxford, and Priory School in Weston-super-Mare.

The syllabus operated on a 100 per cent coursework model for 1988 and 1989; from 1990, following scrutiny by the Schools Examinations and Assessment Council (SEAC), the syllabus was changed to 50% assessment by coursework and 50% assessment by terminal examination. The examination component was split into 25% for a relatively traditional paper and 25% for a more unusual 'process' paper. This process paper consisted of three problem-solving type questions, and candidates' best achievement from two of the questions were used for grading purposes; the criteria for assessment were two of the domains described in the syllabus: Implementation and Interpretation.

Appendix 3b

The Association of Teachers of Mathematics and the Southern Examining Group GCSE Grade Descriptors

Communication

The candidate's ability to present mathematics orally, visually, practically or in continuous prose is judged. There seems to be at least two aspects to this: deciding what to communicate, and how the communication is done.

Aspects likely to be considered:

explaining results; discussing the selection of methods and approaches; stating assumptions; structuring; ensuring clarity using words, diagrams, notation.

Grade F: To be awarded this grade a candidate will be able to explain the problem or situation, and the outcomes of some aspects of the work done. A relatively clear oral, or a brief straightforward account using visual forms of presentation where appropriate may be expected.

Grade C: To be awarded this grade a candidate will be able to present methods and results in an orderly sequence. This might result in a structured report with the candidate taking opportunities to use mathematical symbolisation, graphs, tables etc.

Grade A: To be awarded this grade a candidate will be able to produce a clear, well expressed, structured report in a suitable medium. There might be evidence of discrimination between necessary and unnecessary items, reasons for strategies employed, comments on assumptions and simplifications, effective use of mathematical language, diagrams etc.

Appendix 3b (continued)

Implementation

The candidate's ability to prepare and carry out the task using suitable techniques and skills is judged. These two aspects may occur as distinct stages but may often interact closely with each other.

Aspects likely to be considered:

appraising information given, making initial conjectures, defining sub-problems, simplifying, planning, formulating extensions, accuracy, classifying, specialising, generalising, testing, justifying, experimenting.

Grade F: To be awarded this grade a candidate will be able to make some decisions about how to carry out the tasks and use relatively straightforward techniques. Decisions might be made on the steps to be carried out, and their ordering, the relationships to be established and features to be investigated, what information is needed and where to obtain it. Techniques used may involve exploring by experimenting, using trial and error, testing special cases; using drawings to clarify situations; making appropriate calculations and comparisons.

Grade C: To be awarded this grade a candidate will be able to work systematically and accurately. This may involve deciding what features are to be selected and measured, ordering and categorising, selecting variables; using trial and error methods in a systematic way; enumerating all possible cases; performing calculations. A high degree of accuracy is likely to be shown.

Grade A: To be awarded this grade a candidate will be able to decide on and carry out a task efficiently being aware of the degrees of accuracy involved. This may involve reasoning a plan of approach, determining what might go wrong; ordering information systematically; exhausting all cases and controlling variables; using efficient methods to simplify the task; creating sub-problems and developing different strategies; having flexibility to work in different ways on parts of the task; recognising and using key results to structure further work.

Appendix 3b (continued)

Interpretation

The candidate's ability to see the implications of the work done is judged. This may occur at the end of a task as a summative process, but could equally be inherent in the implementation process.

Aspects likely to be considered:

drawing meaningful conclusions, seeing significance in, making inferences, seeing implications of the work done, investing work with meaning, understanding results, analysing, generalising verifying, justifying, making valid observations about the activity or task, making conjectures, relating results to one another, using different aspects to enhance understanding.

Grade F: To be awarded this grade a candidate will be able to make some valid observations or generalisations about the work. This may involve being able to explain what a number pattern has to do with a particular context.

Grade C: To be awarded this grade a candidate will be able to check results and generalisations. This might involve them in justifying their results for particular cases. They might comment on some of the implications of their results. They could have performed suitable checks on their conjectures and results, perhaps by examining extreme cases. They could analyse data and derive simple formulae.

Grade A: To be awarded this grade a candidate will be able to verify and will attempt to justify conjectures. There could be attempts to provide evidence for the correctness of a conjecture over a range of cases, this might involve simple forms of proof, or the relating of results and conjectures to the starting point of the task. There could be attempts to understand and explain results and conjectures and to gain insight into the situation explored.

Appendix 3b (continued)

Mathematical Attitude

The candidate's appreciation of the possibilities and mathematising are assessed. A candidate might get inside a problem and want to know it from the inside out, being aware of their mathematical abilities and having the confidence to use these appropriately.

Aspects likely to be considered:

engaging in and bringing their mathematical knowledge and experience of a task, involvement, spirit of enquiry, awareness of the possibility of mathematising, appreciation of the relevance and applicability of mathematics to different situations, appreciation of the power of mathematics.

Grade F: To be awarded this grade a candidate will find mathematical activities in relatively straightforward tasks and demonstrate involvement in these. They will be prepared to use suggested resources.

Grade C: To be awarded this grade a candidate will be able to find mathematical activities in less familiar contexts and demonstrate positive involvement in these. They are likely to seek out suitable resources.

Grade A: To be awarded this grade a candidate will seek out and develop mathematical structures to unfamiliar situations. They are likely to demonstrate that the use of mathematics has brought a new understanding to the situations concerned. They are likely to be aware of when further resources are necessary.

Appendix 3b (continued)

Autonomy

The candidate's ability to take responsibility for themselves and the direction of their own learning will be assessed. This could involve making decisions about which activity to work on; how to proceed with it; when to stop; how and when to involve others or to work co-operatively with other; the possible need to find and use other resources.

Aspects likely to be considered:

decision making, independence, accepting responsibility for oneself, directing learning, accepting responsibility for the group, motivating, delegating respecting others, organising.

Grade F: To be awarded this grade a candidate will be able to make decisions about some components of their work. They might decide which problem to start on or how to present their work. They might decide to work by themselves or with others. In a group they could ask for help or information, and are likely to follow a group decision.

Grade C: To be awarded this grade a candidate will be able to make decisions about most aspects of their work.. They are likely to be aware how best they work. They might decide to change their way of working in mid-task, make appropriate decisions about needing information or help. They are likely to take part in group decisions and may accept responsibility for the way a group works.

Grade A: To be awarded this grade a candidate will be able to take most of the decisions involved in their work. This could involve themselves in deciding to work entirely by themselves or sometimes to work with others depending upon the nature of the task. They are likely to have been highly involved in the choice of the task and in making decisions about how to go about it. They could show they are prepared to modify their preferred way of working. In a group they are likely to suggest ways of working and to actively help in reaching decisions.

Appendix 3b (continued)

Evaluation

The candidate's awareness of his/her own work is assessed. This could be part of an on-going process throughout a piece of work or occur as a reflective activity at the end. Both of these should affect what the candidate does in the future.

Aspects likely to be considered:

recognising key results, relating results to existing knowledge, relating results to the problem or sub-problem, commenting critically on methods or results, devising possibilities for further development.

Grade F: To be awarded this grade a candidate will be able to look back over a piece of work and describe some key elements, which can be strategies or results.

Grade C: To be awarded this grade a candidate will be able to look back over a piece of work and describe key elements. In addition, the candidate will be expected to comment critically on the methods employed and the results obtained.

Grade A: To be awarded this grade a candidate will be able to look back over a piece of work as a whole describing critically methods and results. In addition, they should demonstrate that the progress during the work is affected by what has gone on before.

Appendix 4

Comments written by teachers attending a Shropshire InSET course on teaching in mixed-ability groups

Advantages of Setting	Advantages of Mixed Ability	Further thoughts at the end of the day
Teacher A <i>can judge periods of exposition better. (t)</i> <i>can respond more efficiently to individual needs since more individual needs will be <u>group</u> needs (not sure about this). (t & p)</i>	<i>for teaching:</i> <i>able to draw on range of contributions (t)</i> <i>does not pre-judge abilities over the whole range of skills (t)</i> <i>for learning:</i> <i>setting 'right' attitudes to different contributions (p)</i>	<i>Role of teacher; initial stimulus YES group monitoring, discussion facilitator YES, exposition? - ever? when?</i>
Teacher B <i>The able are not held back. (p)</i> <i>Parents want it.</i>	<i>bottom set - bad news. (t & p)</i>	<i>Nothing against principle - if anything in favour. Insecurity because of not being a mathematician. Four hours has not really helped. Concern about how learning is advanced.</i>
Teacher C <i>This section left blank</i>	<i>Introducing certain types of activity, weaker gain from input of brighter ones. (p)</i> <i>Pooling of ideas brainstorming sessions (p)</i> <i>All can contribute all can gain. (p).</i> <i>Working together (p), communicating - oral work (p)</i>	<i>Regular discussions to consider trigger activities; selection of best (ideas)</i> <i>Implementation into particular years focusing on un-setted courseworks in Y10/11 seems to work for us</i> <i>(we operate m-a in years 7/8/9 successfully).</i>

<p>Teacher D <i>Homogeneous groups easier to teach! (t)</i> <i>able to stretch brighter children in groups of similar ability, (t)</i> <i>able to help less able in smaller groups of similar ability (t & p)</i></p>	<p><i>Each child is given the same opportunity. (p)</i> <i>Children are not labelled (p).</i> <i>All children gain from ideas and experiences of other abilities. (p)</i></p>	<p><i>Still wondering how I could manage teaching mixed ability mathematics at 4th/5th year level</i></p>
<p>Teacher E <i>ability range kept to a minimum (kind of) (t)</i></p>	<p><i>Spreads the less able throughout groups (t & p)</i> <i>Doesn't categorise pupils (p)</i></p>	<p><i>appreciation of "moral" issue.</i> <i>ideas for motivating the kids.</i> <i>Good starting points</i></p>
<p>Teacher F <i>This section left blank</i></p>	<p><i>Does not disillusion pupils by putting them in the bottom set in Y7. (p)</i></p>	<p><i>Mixed ability does have a place in the upper school, It's a case of teaching style, getting pupils used to way of working and using, amongst other things a suitable starting point Setting limits children's access to the realisation of their potential - (sweeping generalisation?),</i></p>
<p>Teacher G <i>Easier to cover topics leading to public exams (t)</i> <i>Less spread of ability means class can work together on topics. (p)</i> <i>Easier classroom management (t).</i> <i>Homework setting and assessment (t)</i> <i>Less ground to cover easier use of resources (t)</i></p>	<p><i>Each pupil can move faster slower as appropriate (p), levelling up (p), no upheaval with moving from set to set (p), equal access (p), opportunity to develop (p), not labelling (p), new chance for Y7 (p) diversity of earlier experience different responses to topics - good at some things (p), co-operation (p), more open teaching (t) investigative approaches (p)</i></p>	<p><i>Potentially good but requires a <u>different mind set</u> - work must be prepared with this in mind... it's no good just mixing up kids into different groups .</i> <i>I have a clearer idea now that one starting point is to choose a good 'starter' for the topic I want to cover</i></p>

Glossary of terms

Access and accessible tasks

These are problems, puzzles and starting points to lessons that all children can begin working on, and which require the learner to have only minimal prior knowledge in order to proceed.

Examples of such tasks are:

- playing a game of four-in-a-line with pegs and pegboards - to develop ideas of co-ordinates, sequences and graphs;
- making cuboids with multi-link - to develop ideas of dimensions, surface area and volume;
- paper folding - to develop ideas of symmetry, tessellation, angle and formulae;
- think of a number puzzles - to develop ideas of solving equations

Answering pupils' questions

The type of answer I offer a pupil will depend upon my knowledge or my perception of a child's need at a specific time. The implication of this is I may offer a different answer or response to different pupils asking the same question. For example whereas I may remind one pupil of the convention of the system of co-ordinates, I may ask another to explore what happens when the co-ordinates are plotted (y, x) instead of (x, y) .

Atomisation of skills

This describes pupils repeatedly carrying out a certain type of calculation, such as adding decimals, or a procedure, such as solving equations, in isolation to other skills or concepts. A number of similar, short questions requiring specific answers would epitomise an atomised approach.

BCME conferences

The British Congress of Mathematics Education conference is a bi-annual event. The first BCME conference took place at Loughborough (1991), the second at Leeds (1993) and the third at Manchester (1995).

Challenge

Providing pupils puzzles and problems to solve is a fundamental underpinning of the way I teach mathematics. Such problems may have 'fixed' answers, such as the effect of m and c in $y = mx + c$, or they may have more 'open' answers, such as the number of different ways of arranging six dots on a page.

Classifying triangles and quadrilaterals

Small groups of pupils are given 9 geo-strips, 3 each of three different lengths; L (long), M (medium) and S (short). Sets of strips are chosen so that certain groups would not be able to make the complete set of 10 triangles, i.e. LLL, MMM, SSS, LLM, LLS, MML, MMS, SSL, SSM, LMS. i.e. if $2S < M$, or $2S < L$ or $2M < L$ or $S + M < L$, then certain of the 10 triangles cannot be made. The extension task is to consider the different quadrilaterals that can be made by pairing up properties of the number of Right Angles with Pairs of Parallel Sides. Once a complete set has been found, these can be matched with the number of Equal Sides, and a complete set can then be searched for.

Content skills

These are described in attainment targets 2, 3 and 4 in National Curriculum.

Curriculum development

This could be a change in the way I teach a particular lesson, developing ideas for a new module, or re-writing an entire scheme of work. I consequently define curriculum development as small changes in my practice which influence my wider view of the way I organise and teach the curriculum.

Dance Squared

This is a motion geometry picture produced by the Canadian Film Board. It illustrates a number of ways a square can be dissected into smaller pieces which are re-formed into different shapes. In the main the original square is dissected into isosceles right angled triangles, smaller squares and rectangles.

Differentiation

This describes children gaining different understandings and achieving different outcomes from a common starting point within the same class.

My definition of differentiation is based upon articles from MT152: pp.26-36.

Entitlement

This is each pupil's right to be provided with access to learning the body of knowledge as enshrined in National Curriculum Mathematics.

Entry-point questions

These are intentionally easy questions which provides each person in a class with an opportunity to make a response.

Equality

Throughout this thesis my focus equality is related to children's learning opportunities. Whilst issues pertaining to "race" and gender are fundamentally important, I have chosen not to explore them; instead I have considered equality from the perspective of making mathematics accessible to all children.

Extension or next tasks

These are ideas or problems which I offer learners to further develop their mathematical thinking. Many extension tasks are pre-planned and I decide when it is appropriate to suggest an idea to an individual or to a group according to my perception of need at particular times. Other extension tasks will be unplanned and arise as a natural consequence of the work pupils do.

Exploration

This is the process of pupils finding out how certain mathematical structures work. For example: exploring how the equation of a line changes under a transformation of rotation through 90° , 180° and 270° about $(0, 0)$. I interchange 'exploration' and 'investigation'.

Families of graphs

These are parallel linear graphs.

Fractions and decimals worksheet

Below is a copy of a sheet I produced for a Y9 class to encourage pupils to develop further their understanding of fractions and decimals. My intention was to offer a variety of ideas for pupils to develop and explore, having worked in an earlier lesson on converting fractions to decimals.

Fractions and Decimals

I want you to work on and DEVELOP some of these problems over the next few lessons.

Look at your chart and write about any of the patterns that you notice, See if you can predict some results for a larger chart without using a calculator,

Look at all the decimal answers that have an answer of 0.5

Write down all the fractions that give this result, Join them together with a straight line, Try to predict some other fractions that would also give a decimal answer of 0.5

Now do the same for 0.25

Now look for some other results that can be joined together to form straight line graphs. Write about the different steepness of the graph lines.

Add the decimal answers for $\frac{1}{2}$ and $\frac{1}{3}$ together. Now look for the same decimal answer on your chart and see what the fraction for this is.

Now try adding some other pairs of decimal answers for different fractions together and see if you can find what the decimal answer would be as a fraction, When you have done five or six of these see if you can find any rules for adding fractions.

Try some subtractions

Look at the decimal conversion of a certain fraction and put this on your calculator, Now press the $\frac{1}{x}$ key. Write down your result and notice where it appears on your chart in comparison to your starting value. Do a lot of these types of calculations and write about what you notice.

Fragmented skill acquisition

Typical of this style is when a teacher provides a procedure to carry out certain, specific calculations, (e.g. how to write numbers in standard index form) whereupon pupils practise the procedure by doing an exercise from a worksheet or a text book. Skills are therefore taught in isolation to one another.

Geo-strips

These are plastic strips of various lengths and colours. They can be connected together using split pins.

Geo-boards (9-pin)

These are square boards (approximately 20cm x 20cm) with 9 pins or nails in a 3 by 3 array. Shapes are made by connecting pins together with an elastic band.

Grid Algebra

This idea, devised by Hewitt, D. (University of Birmingham), is based upon numbers written on a grid in the arrangement of a multiplication square. Movements, from one place to another on the grid, are explored as functions; arising from which, algebraic statements are constructed.

Holism and holistic approaches

This determines how I plan schemes of work so that the same skills are met in a variety of contexts and within one content a variety of skills can be developed. For example, fractions are met in contexts such as shape and space paper-folding problems, calculations of area in enclosure-type problems and in probability experiments. Furthermore, whilst pupils are working in any of these contexts, they will also meet and draw upon other skills. For example, when working with shape and space paper-folding problems, pupils will engage with: properties and names of shapes, angles of shapes, angle sums of shapes (therefore addition), areas of shapes, perimeters of shapes (therefore Pythagoras' theorem) and constructing formulae.

In-depth knowledge

This is the exploration of concepts which transcends knowledge based upon tricks, algorithms or rules, which the learner may be able to repeat yet does not understand. For example, knowing that the sine of an angle is "the opposite over the hypotenuse" is different to knowing why this is true and how such a rule can be constructed.

Inter-connected approaches

I view the curriculum as a web of connected skills and concepts. Each skill and concept exists in and can, therefore, be recognised within various contexts.

Investigative approaches

This describes an approach which permeates pupils learning of mathematics where skills and concepts are met through learners solving problems and exploring situations. I draw a distinction between investigative approaches and "doing an investigation". This latter approach is one where the skills and concepts pupils may use are not exclusively pre-determined by the starting point. Finding a problem for exploration with the intention of aiding pupils conception of, say, Pythagoras is an example of an investigative approach being applied directly to the content.

IRATs problem (Isosceles right angled triangles)

This problem is one I first came across whilst studying the Open University ME234 course in 1989. It involves folding and cutting isosceles right angled triangles in the line of symmetry. After one-fold and a cut, there are two half size irat's. After two-folds and a cut we have two quarter size irats and a square which is half the area of the original triangle.

Knowledge and wisdom

My attention was drawn to consider the difference between knowledge and wisdom during an OUCME research day, when Mason offered the following:

Knowledge:- knowing that, knowing how. Wisdom:- knowing to.

I subsequently wrote a short paper which I have produced below:

Within a particular context, deciding how and when to use and apply knowledge in ways appropriate to the context is a shift from having knowledge to having wisdom. Mason's description of wisdom fits my understanding of using and applying mathematics i.e. having wisdom is to know when it is appropriate to use knowledge and apply a particular skill.

I shall describe a scene from the film 'Butch Cassidy and the Sundance Kid' to illustrate the difference between knowledge and wisdom. The scene takes place when Butch and Sundance have just been hired as payroll guards by Percy Garrett, a mine owner. The three of them are riding down the mountain in order to pick up the payroll from the bank. The following dialogue takes place:

Butch:	"I think they're in the trees up ahead."
Sundance:	"In the bushes on the left."
Butch:	"I'm telling you, they're in the trees up ahead."
Sundance:	"You take the trees, I'll take the bushes."

At this point an exasperated Percy turns to Butch and Sundance -

Percy:	"Will you two beginners cut it out."
Butch:	"Well we were just trying to stop an ambush Mr Garrett."
Percy:	"Morons! I've got morons on my team. Nobody is going to rob us going down the mountain - we have got no money going down the mountain. When we have the money, on the way back, then you can sweat. Bingo."

Whilst Butch and Sundance had knowledge about how and where to look for an ambush, they seemingly lacked the essential wisdom of knowing when to apply this knowledge.

Lead lessons

With the agreement of colleagues, I would lead a team-taught lesson with their classes. This input would form the basis upon which the pupils then proceeded to work for a sequence of further lessons.

Mathematical:

- **concepts**

This is a collection of narrower skills (see below). For example to gain a concept of Trigonometry, the learner will need to know about skills such as measuring distances and angles.

- **procedures**

This is knowing what steps to take to solve a particular problem. For example knowing the steps involved in working out the length of a hypotenuse of a right-angled triangle using certain other information.

- **skills**

An example of a skill is being able to use a protractor to measure an angle.

- **structures**

An example of a structure is the recognition of how trigonometric ratios can be represented in graphical form and how such graphs change under certain conditions, i.e. $2 \sin x$, $\sin 2x$ etc.

Module

A module is a collection of ideas based upon a starting point and extension tasks. The main concept being identified by the title, e.g. 'Transformations'. At Orleton Park School, in Y7 Y8 and Y9, a module was timetabled for two to three weeks of curriculum time. In Y10 and Y11 a module lasted for a minimum of three weeks and could extend, in some circumstances to six weeks.

Modularised curriculum framework

A timetable for each year group provided information for staff to plan when to teach each module. This framework offered cohesion and avoided resources and equipment being required simultaneously by different members of staff.

Oaktree School

This a non-fee paying private school in Liverpool. Pupils attended Oaktree because they had left mainstream schools for a variety of reasons such as truancy and behaviour.

Orleton Park School

Orleton Park was a small 11-16 mixed comprehensive in North Telford (550 on role). From 1991 following the demise of catchment areas, owing to the three closest neighbouring schools gaining grant maintained status, we worked hard to keep pupil numbers reasonably stable. Pupils came from a wide range of backgrounds, travelling from all regions in and around Telford. There was no 'captive' feeder school and the intake came from 15 primary schools; one of the larger feeder primary being 5 miles away. In 1992 21.6% of pupils in the school took free school meals this increased to 25.0% by 1994. 6% of pupils did not have English as their first language. There was a Moderate Learning Difficulty Unit attached to the school, catering for 31 statemented pupils. These pupils were integrated into mainstream classes wherever possible.

Open University CME research days

These are termly gatherings of invited speakers researchers and supervisors who meet together under the auspices of the Centre for Mathematics Education at the Open University to disseminate and discuss on-going or completed research in mathematics education.

Palindromic Numbers

These are numbers that read the same backwards as they do forwards e.g. 4884.

Paper folding

I use paper folding activities to develop concepts of 2-d and 3-d shape and space. For example, with 2-d work, I frequently begin with A₆ paper and using one, two or three folds it is possible to create a range of polygons. This subsequently opens up opportunities to develop concepts of properties and names of shapes, angle sizes, tessellations and further work on areas and perimeters of shapes.

Pentominoes

A pentomino is a 2 dimensional shape which made by joining together five equal sized squares, by their edges. Excluding rotations and reflections, there are 12 such shapes.

People-maths problems

These are problems where pupils/students take an active role in working towards a solution. Most people-maths problems requires the learners to work together, often without sitting behind desks.

Pre-determining outcomes

Offering different pupils different tasks at the beginning of each module or each lesson is to pre-determine pupils' learning outcomes.

Problem solving approaches

Problem solving in my classroom would be typified by posing the following question: *How many different quadrilaterals can be made on a 9-pin geoboard.* As pupils proceed with this task there exist a wide variety of other skills to be developed, practised or consolidated (dependent upon each pupil's current mathematical knowledge), i.e.. geometrical properties, names of shapes, areas, symmetries. Other problems such as classifying the shapes and trying to show that a complete set had been found can also be pursued.

Principles and strategies

The most significant principle upon which I base my practice is working with non-selective, un-setted groups. The strategies are the methods I use to support this principle.

Resources

This term covers a wide range of types of equipment such as:

- different types of grid paper (e.g. squared, iso-metric);
- drawing implements (e.g. rulers and compasses);
- manipulatives (e.g. geo-boards, multi-link cubes and Cuisenaire rods);
- calculators (e.g. simple, scientific, graphic), and computers.

Routes on a Grid

This idea is described in: Banwell, C., Saunders, K. and Tahta, D. (1972) *Starting Points*, Oxford University Press. Pupils explore routes on a 16-dot square grid. A possible outcome leads pupils to engage with Pascal's Triangle.

Simple starting points

These are the collection ideas, tasks and problems that I use in order to engage learner's interest to cause then to develop mathematical concepts. A typical starting point task, in order to develop pupils' shape and space concepts is using a 9-pin geoboard to find triangles and quadrilaterals.

Snook

This is the name for a problem which has become widely used. The problem is based upon a journey made by a 'snooker ball' on a square grid. It involves the exploration of a the number of features such as how many grid squares the ball passes through as it follows a diagonal path which begins at one corner and rebounds from the sides until it reaches another corner.

Story Lines

These describe my skeleton plan for teaching each topic in the modular framework. A story line describes the starting task together with the extension ideas or next tasks. A typical example, using 'Trigonometry' as the topic, is:

- constructing rotating arm;
- H (horizontal) and V (vertical) measures for each 10° angle up to 90° ;
- analysing result and considering when $H = V$;
- checking for accuracy using cos and sin keys;
- predicting results for 90° to 360° ;
- graphing results;
- working out H and V lengths for rotating arms other than unit length;
- engaging with the vocabulary of hypotenuse, opposite, adjacent;
- drawing own diagrams on plain paper, measuring an angle and the hypotenuse, calculating H and V lengths then measuring with a ruler to check accuracy;

- working out angles if hypotenuse and adjacent or hypotenuse and opposite are known, again drawing own diagrams to carry out this task
- working out hypotenuse when angle and the opposite or adjacent is known
- practical applications - clinometres;
- trig. calculations in 3-D (diagonal through multi-link cuboids);
- considering $\sin^2 x + \cos^2 x$.

Story Posters

These are posters that I write and display along with the pupils work. A story poster is designed as an aid-memoire, to provide information as a brief resume of the main ideas discussed in a topic. Some posters will relate specific conversations or outcomes of certain events.

Subordinate Skills

These are skills which underpin the development of more complex concepts. For example being able to carry out the skill of addition is subordinate to the concept of working out a mean average. Likewise being able to work out a gradient is subordinate to differential calculus.

Teacher interventions

An intervention will depend upon my perception of different pupils' needs. Deciding how much to tell a pupil something or whether to stand back and suggest a pupil thinks something through for themselves is at the heart of my interventions.

Teacher lust

This describes the actions of a teacher who claims power and authority over the learners, who in turn become answerable to that teacher. See Tahta [1972:11].

Teacher notes

These are notes which describe starting points, extension tasks and useful resources for each module. Teacher notes, for each year group, were placed in A4 plastic wallets and collected together into clip files. Notes were up-dated each year.

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Telephone Cubes

The pupils sit back to back. One pupil make a shape from (say) 7 cubes then attempts to describe it to a partner who has to follow the instructions and attempt to make the same shape.

The South Nottinghamshire Project

I have taken my description of this from Schools Council [1977: pp.140-141]

Mixed-ability teaching in mathematics:

This project accepts the principle of the whole class working in the same general field but with differentiation of the particular problems attempted by individuals. It rejects individualized, independent learning schemes as a strategy for teaching mixed-ability classes

The Pyramids work, which is taken from this project, creates opportunities for pupils to solve missing number type problems by constructing equations and then solving them.

Trust analogy

In Shrewsbury there is a whole food shop called Crabapple. In one corner of the shop there are a multitude large jars containing herbs and spices, a set of weighing scales, some paper bags hanging on a string, a pen and a sellotape dispenser. Customers weigh out their own herbs and spices and write the cost on the paper bag according to the price per ounce shown on each jar. I enjoyed shopping there because of the trust the proprietors exhibit. One day I discussed this with the proprietors; we agreed that trust was possibly the most important quality that people can show towards each other.

$$y = mx + c$$

This is an exploration of graphs of linear functions. Pupils investigate the effect of varying m and c on the graphs produced.